



SI 101 S-parameters

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Z and Y Matrices (1)

$$\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

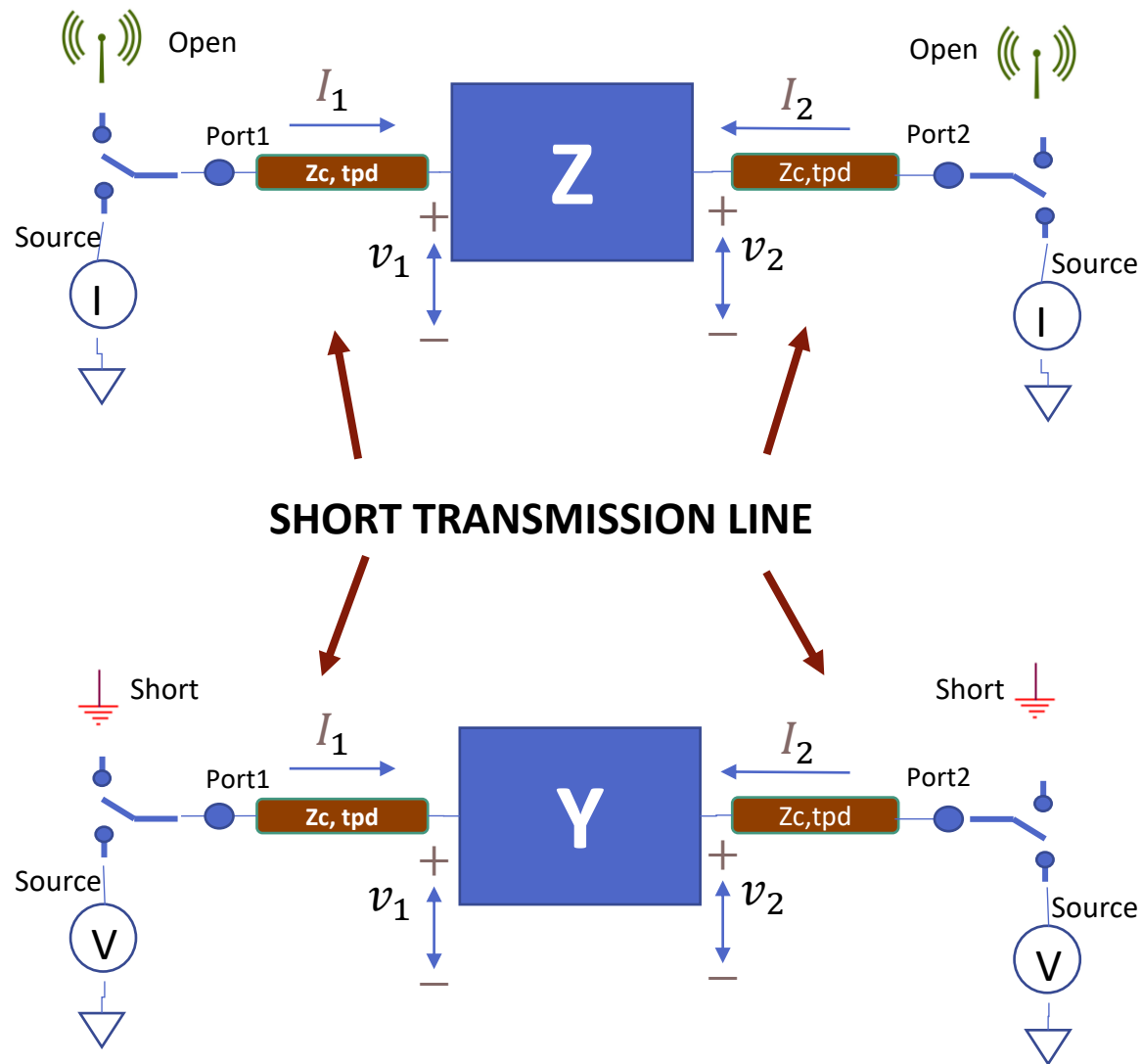
$$Z_{11} = \left. \frac{v_1}{I_1} \right|_{I_2=0} \quad Z_{12} = \left. \frac{v_1}{I_2} \right|_{I_1=0}$$

$$Z_{21} = \left. \frac{v_2}{I_1} \right|_{I_2=0} \quad Z_{22} = \left. \frac{v_2}{I_2} \right|_{I_1=0}$$

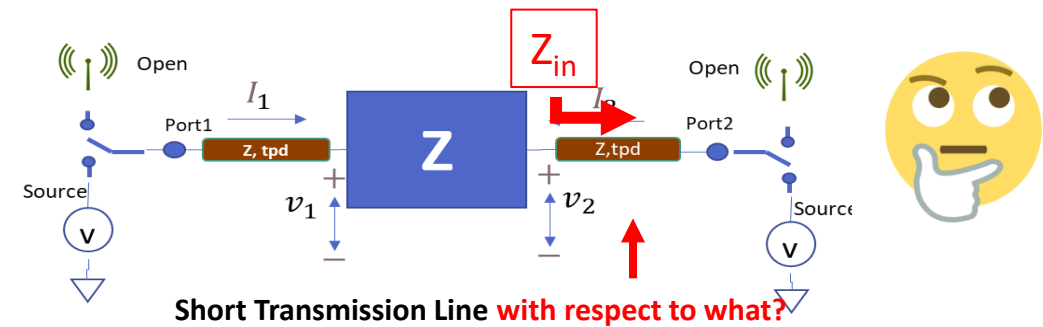
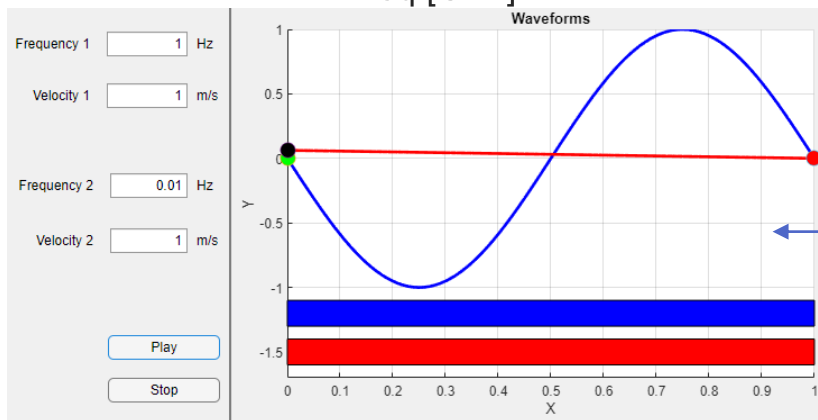
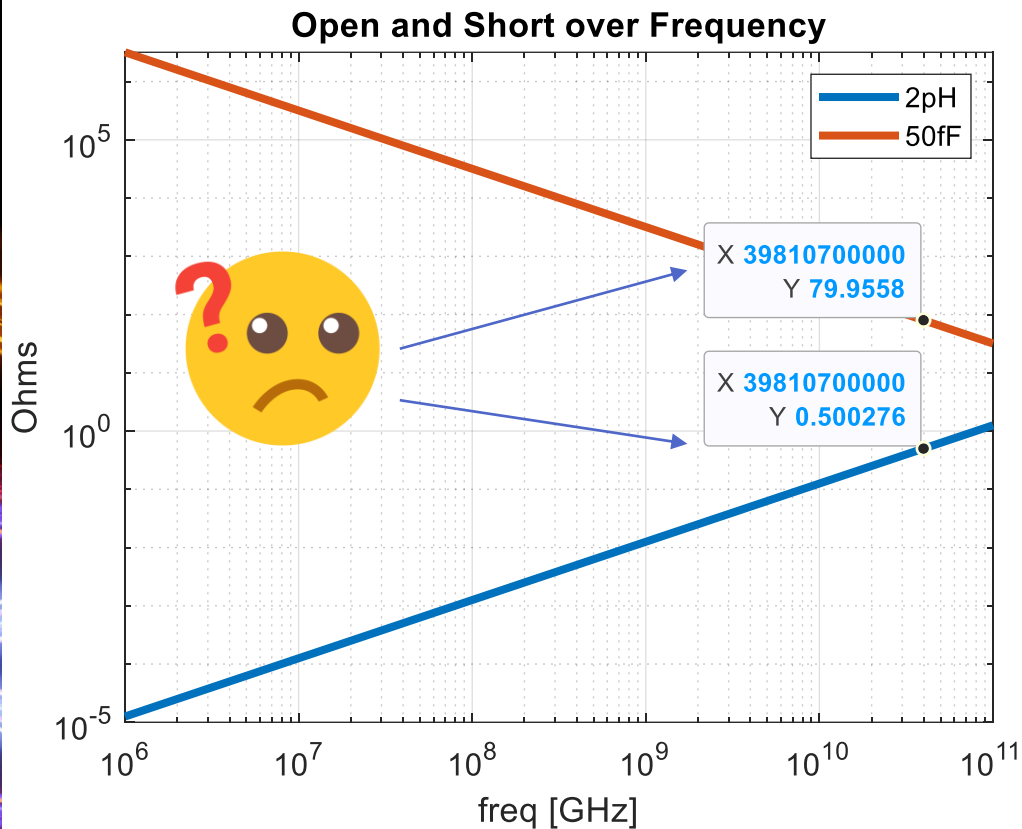
$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

$$Y_{11} = \left. \frac{I_1}{v_1} \right|_{v_2=0} \quad Y_{12} = \left. \frac{I_1}{v_2} \right|_{v_1=0}$$

$$Y_{21} = \left. \frac{I_2}{v_1} \right|_{v_2=0} \quad Y_{22} = \left. \frac{I_2}{v_2} \right|_{v_1=0}$$

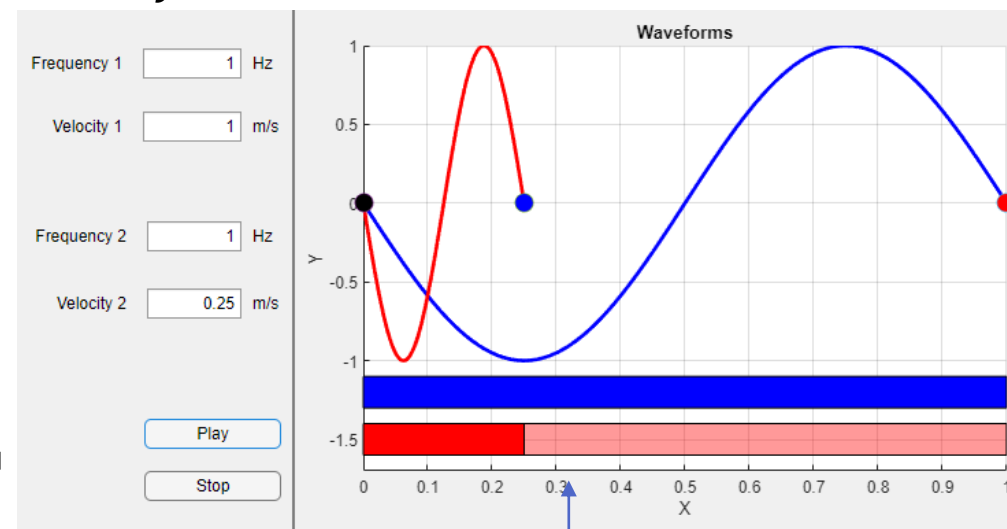


Z and Y Matrices (2)



$$\lambda = \frac{v}{f} \text{ The Wave-Length}$$

$$z_{in} = z_0 \cdot \frac{z_L + jz_0 \tan \beta d}{z_0 + jz_L \tan \beta d}$$



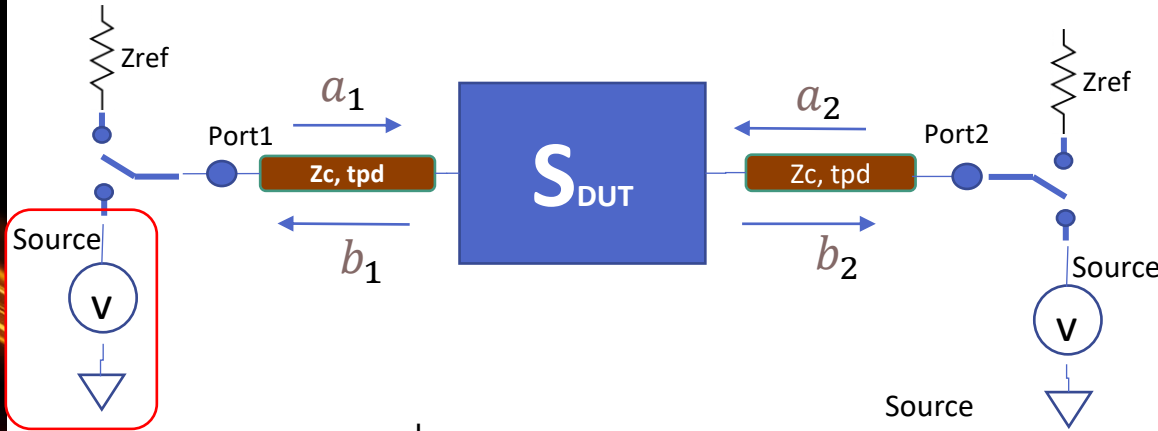
Same frequency, different propagation speed

Speed = Speed/4, electrically $L_{\text{length}} = L_{\text{length}}$

For the same length, different frequency, the red is considered lumped, since $L = \lambda/100$

λ is our measurement stick

S-Parameters



$$a_x = \frac{v_x^+}{\sqrt{Z_{ref}}} \quad b_x = \frac{v_x^-}{\sqrt{Z_{ref}}}$$

$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$$

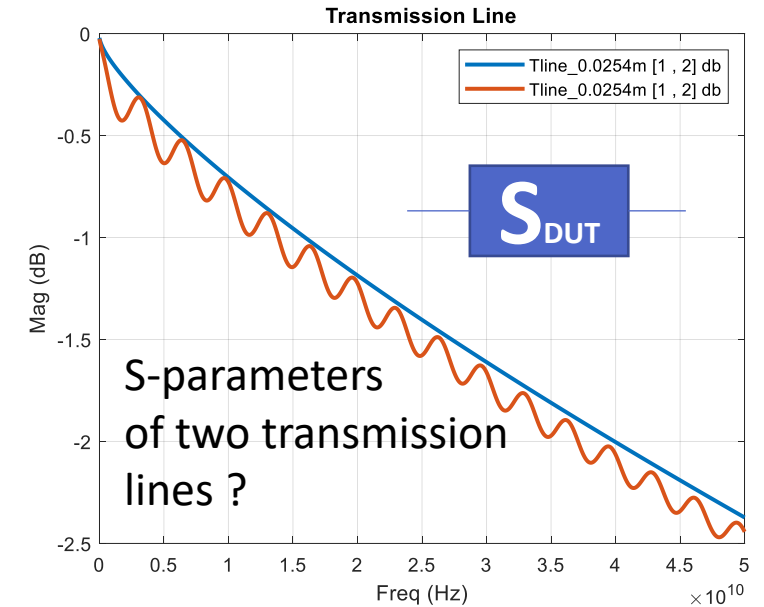
$$S_{11} = \left. \frac{b_1}{a_1} \right|_{a_2=0} \quad S_{12} = \left. \frac{b_1}{a_2} \right|_{a_1=0}$$

$$S_{21} = \left. \frac{b_2}{a_1} \right|_{a_2=0} \quad S_{22} = \left. \frac{b_2}{a_2} \right|_{a_1=0}$$

- Independent of transmission line length (mostly is just a delay)
- Easier to terminate to an impedance not an open/load)
- All the small variations can be calibrated out

$$\Gamma = \frac{Z_{REF} - Z_C}{Z_{REF} + Z_C} = 0 \Rightarrow Z_{REF} = Z_C$$

Means, no reflections at the port (where the instrument is)

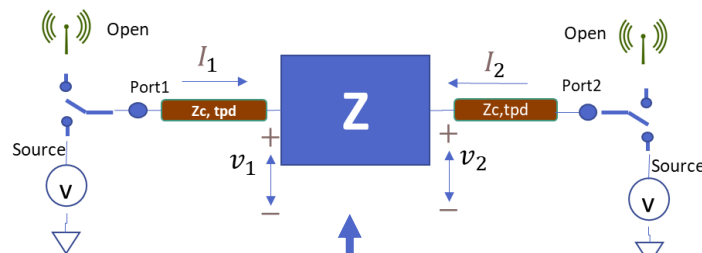


NO!!!, $Z_{REF} = 50 \text{ Ohms}$, $Z_{REF} = 40 \text{ Ohms}$

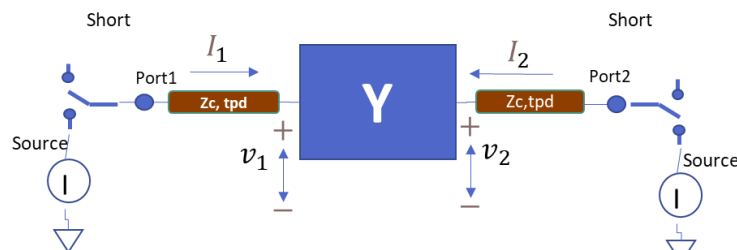
<https://www.signalintegrityjournal.com/articles/270-s-parameter-renormalization-the-art-of-cheating>

S PARAMETERS ARE INCOMPLETE WIHTOUT ZREF

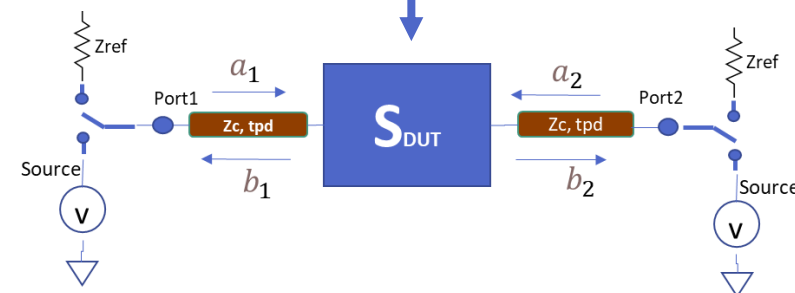
Timeline



Very low frequency



Easier to get a short than an open, extended its use to higher frequencies



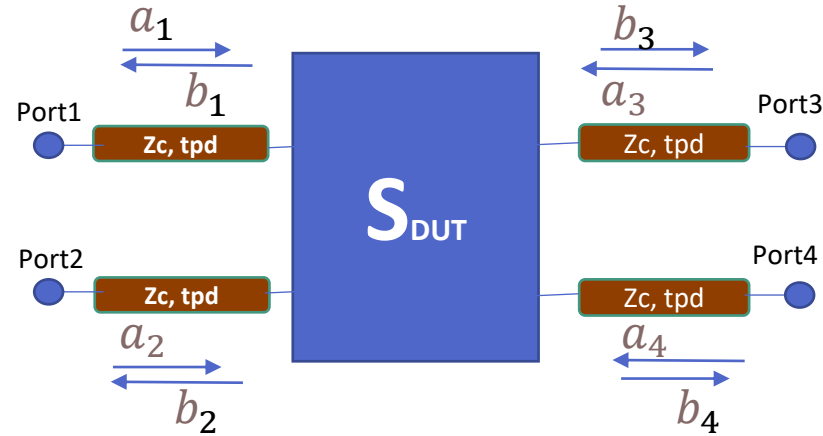
S_{11}	S_{11}	$\frac{(Z_{in} - Z_0)(Z_{01} + Z_0) - Z_{01}Z_{02}}{(Z_{in} + Z_0)(Z_{01} + Z_0) - Z_{01}Z_{02}}$	$\frac{(Y_1 - Y_0)(Y_0 + Y_{01}) - Y_{01}Y_{02}}{(Y_1 + Y_0)(Y_0 + Y_{01}) - Y_{01}Y_{02}}$	$\frac{A + BZ_0 - CZ_0 - D}{A + BZ_0 + CZ_0 + D}$
S_{12}	S_{12}	$\frac{2Z_0Z_{01}}{(Z_{in} + Z_0)(Z_{01} + Z_0) - Z_{01}Z_{02}}$	$\frac{-2Y_0Y_{01}}{(Y_1 + Y_0)(Y_0 + Y_{01}) - Y_{01}Y_{02}}$	$\frac{2(AD - BC)}{A + BZ_0 + CZ_0 + D}$
S_{21}	S_{21}	$\frac{2Z_0Z_{02}}{(Z_{in} + Z_0)(Z_{01} + Z_0) - Z_{01}Z_{02}}$	$\frac{-2Y_0Y_{02}}{(Y_1 + Y_0)(Y_0 + Y_{01}) - Y_{01}Y_{02}}$	$\frac{2}{A + BZ_0 + CZ_0 + D}$
S_{22}	S_{22}	$\frac{(Z_{in} + Z_0)(Z_{02} - Z_0) - Z_{01}Z_{02}}{(Z_{in} + Z_0)(Z_{01} + Z_0) - Z_{01}Z_{02}}$	$\frac{(Y_1 + Y_0)(Y_{02} - Y_0) - Y_{01}Y_{02}}{(Y_1 + Y_0)(Y_0 + Y_{01}) - Y_{01}Y_{02}}$	$\frac{-A + BZ_0 - CZ_0 + D}{A + BZ_0 + CZ_0 + D}$
Z_{in}	Z_{in}	$Z_0 \frac{(1 + S_{11})(1 - S_{22}) + S_{12}S_{21}}{(1 - S_{11})(1 - S_{22}) - S_{12}S_{21}}$	$\frac{Y_{01}}{Y_{01}Y_{02} - Y_{02}Y_{01}}$	$\frac{A}{C}$
Z_{out}	Z_{out}	$Z_0 \frac{2S_{21}}{(1 - S_{11})(1 - S_{22}) - S_{12}S_{21}}$	$\frac{-Y_{02}}{Y_{01}Y_{02} - Y_{02}Y_{01}}$	$\frac{AD - BC}{C}$
Z_{in}	Z_{in}	$Z_0 \frac{2S_{11}}{(1 - S_{11})(1 - S_{22}) - S_{12}S_{21}}$	$\frac{-Y_{01}}{Y_{01}Y_{02} - Y_{02}Y_{01}}$	$\frac{1}{C}$
C_{in}	C_{in}	$Z_0 \frac{(1 - S_{11})(1 + S_{22}) + S_{12}S_{21}}{(1 - S_{11})(1 - S_{22}) - S_{12}S_{21}}$	$\frac{Y_{01}}{Y_{01}Y_{02} - Y_{02}Y_{01}}$	$\frac{D}{C}$
Y_{in}	Y_{in}	$Y_0 \frac{(1 - S_{11})(1 + S_{22}) + S_{12}S_{21}}{(1 - S_{11})(1 - S_{22}) - S_{12}S_{21}}$	$\frac{2Z_0}{Z_{01}Z_{02} - Z_{02}Z_{01}}$	$\frac{D}{B}$
Y_{out}	Y_{out}	$Y_0 \frac{-2S_{21}}{(1 + S_{11})(1 + S_{22}) - S_{12}S_{21}}$	$\frac{-Z_{01}}{Z_{01}Z_{02} - Z_{02}Z_{01}}$	$\frac{BC - AD}{B}$
Y_{in}	Y_{in}	$Y_0 \frac{2S_{11}}{(1 + S_{11})(1 + S_{22}) - S_{12}S_{21}}$	$\frac{-Z_{02}}{Z_{01}Z_{02} - Z_{02}Z_{01}}$	$\frac{-1}{B}$
Y_{out}	Y_{out}	$Y_0 \frac{(1 + S_{11})(1 - S_{22}) + S_{12}S_{21}}{(1 + S_{11})(1 + S_{22}) - S_{12}S_{21}}$	$\frac{Z_{01}}{Z_{01}Z_{02} - Z_{02}Z_{01}}$	$\frac{A}{B}$
A	A	$\frac{(1 + S_{11})(1 - S_{22}) + S_{12}S_{21}}{2S_{21}}$	$\frac{-Y_{01}}{Y_{01}}$	A
B	B	$\frac{Z_0 \frac{(1 + S_{11})(1 - S_{22}) + S_{12}S_{21}}{2S_{21}}}{2S_{21}}$	$\frac{-1}{Z_{01}}$	B
C	C	$\frac{1}{Z_0} \frac{(1 - S_{11})(1 - S_{22}) - S_{12}S_{21}}{2S_{21}}$	$\frac{Y_{01}Y_{02} - Y_{02}Y_{01}}{Y_{01}}$	C
D	D	$\frac{(1 - S_{11})(1 + S_{22}) + S_{12}S_{21}}{2S_{21}}$	$\frac{-Y_{02}}{Y_{01}}$	D

Then, in simulation world mathematically, we could compute any N-port parameters from any other.

With advanced Calibration methods, 100+ GHz and beyond

Generic Timeline

N-Ports & Mixed Mode S-Parameters



$$b_1 = s_{11}a_1 + s_{12}a_2 + s_{13}a_3 + s_{14}a_4$$

$$b_2 = s_{21}a_1 + s_{22}a_2 + s_{23}a_3 + s_{24}a_4$$

$$b_3 = s_{31}a_1 + s_{32}a_2 + s_{33}a_3 + s_{34}a_4$$

$$b_4 = s_{41}a_1 + s_{42}a_2 + s_{43}a_3 + s_{44}a_4$$

$$\frac{b_4 - b_3}{a_2 - a_1} = \frac{b_{d2}}{a_{d1}}$$

$$a_1 = -a_2 \Rightarrow a_{d1} = 2a_2$$

Boundary condition

It's all about
boundary
condition setup

$$a_{1,3} = -a_{2,4}$$

$$a_{1,3} = a_{2,4}$$

$$a_d = a_{1,3} - a_{2,4}$$

$$a_d = a_{1,3} + a_{2,4}$$

$$b_{1,3} = -b_{2,4}$$

$$b_d = b_{1,3} - b_{2,4}$$

S_{DD}

S_{DC}

$$b_{1,3} = b_{2,4}$$

$$b_d = b_{1,3} + b_{2,4}$$

S_{CD}

S_{CC}

$$b_3 = -s_{31}a_2 + s_{32}a_2$$

$$b_4 = -s_{41}a_2 + s_{42}a_2$$

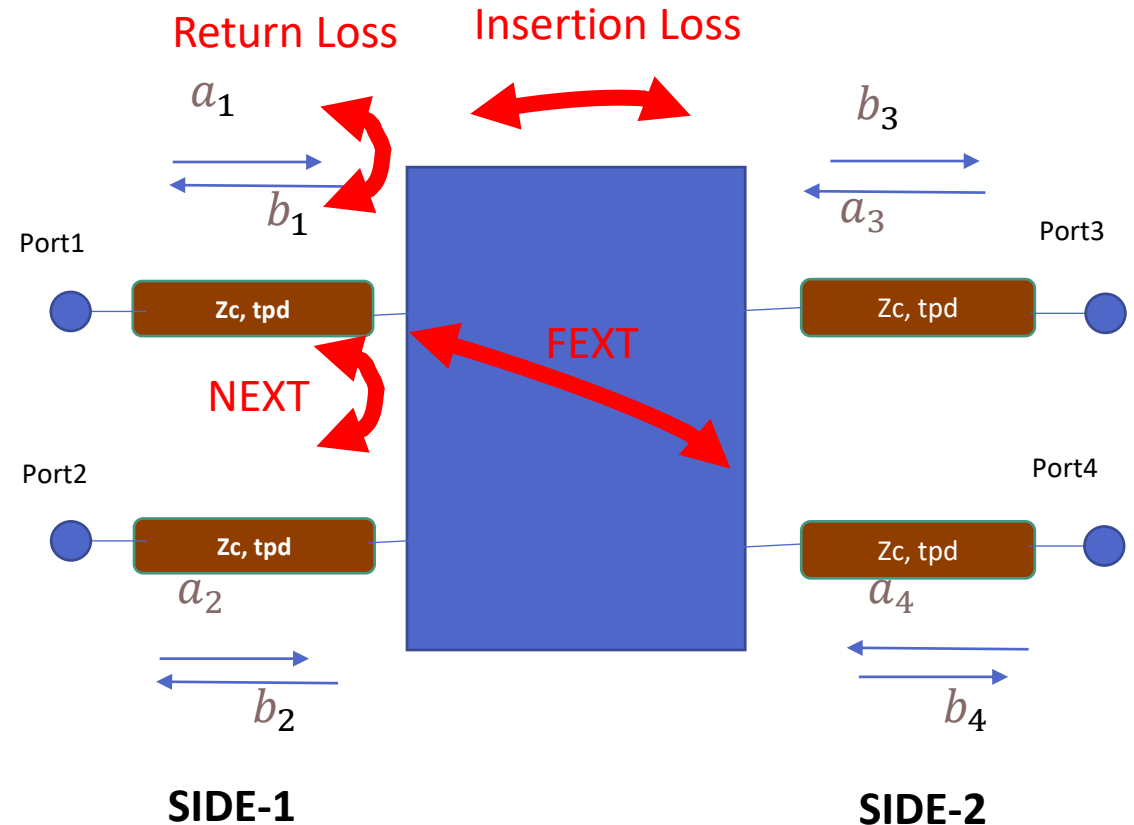
$$b_4 - b_3 = b_{d2} = 0.5 a_{d1} (-s_{41} + s_{42} - s_{31} + s_{32})$$

$$s_{dd21} = \frac{b_{d2}}{a_{d1}} = 0.5 (-s_{41} + s_{42} - s_{31} + s_{32})$$

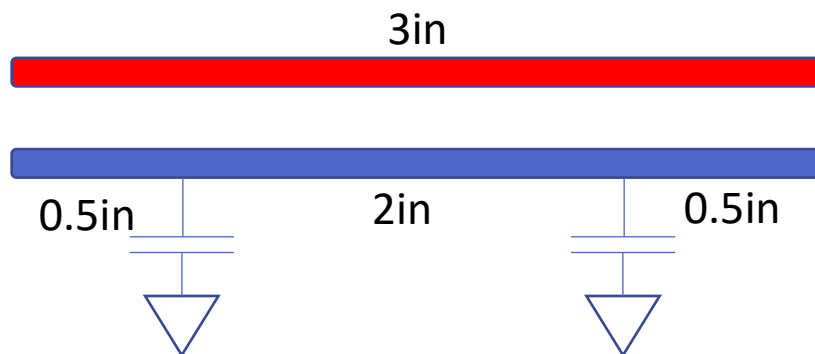
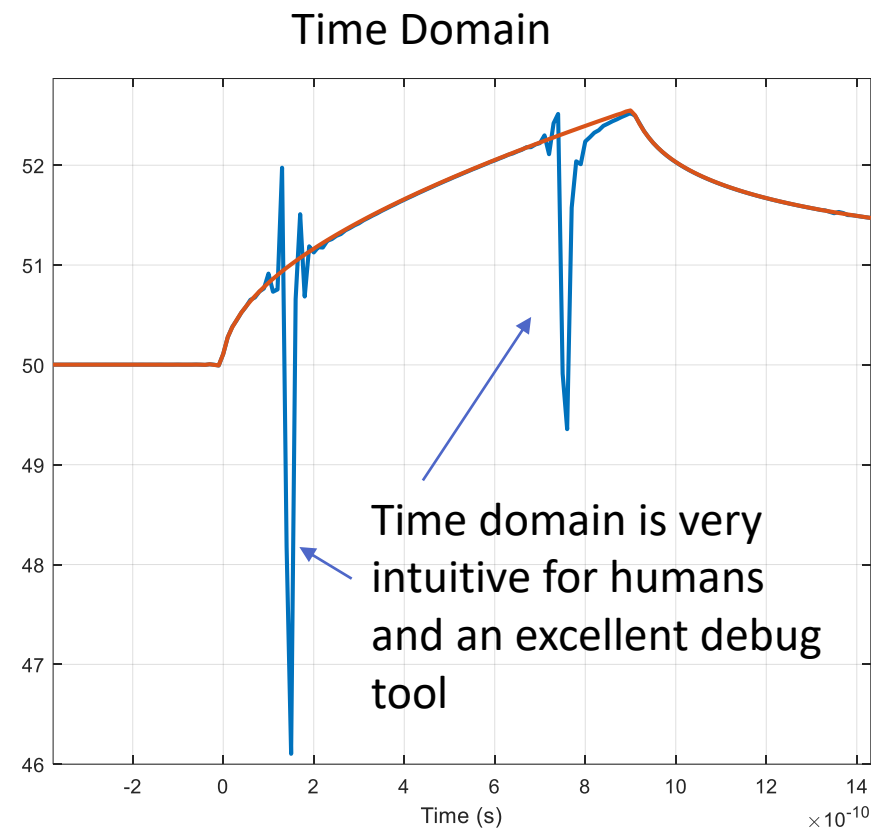
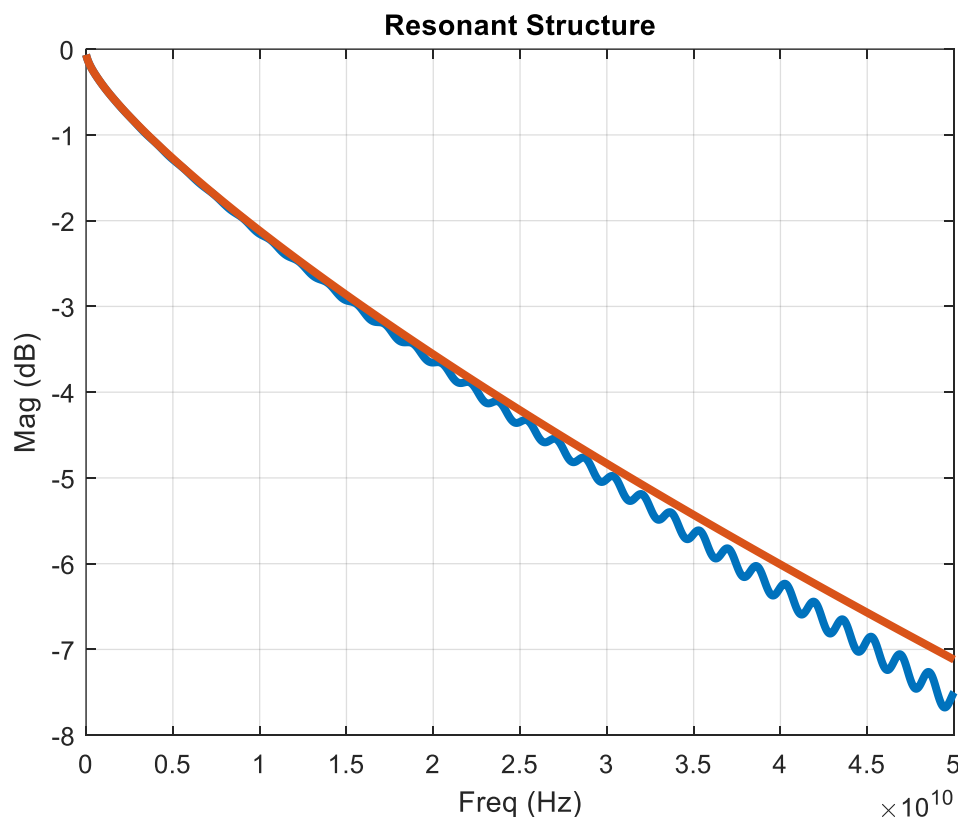
Differential Insertion
Loss

Common Nomenclature

- **Return Loss:** Like reflection in SI, means all that return is lost since it does not get to the other side
- **Insertion Loss:** When we insert something (for passives), on a perfect channel we incur losses. When we have actives, in general this is called Gain (G), but it's the same thing
- **Crosstalk:** Energy diverted to a non intended port (receiver)
 - **Far end crosstalk or FEXT:** When the “victim” port is on the far side of the “aggressor” port
 - **Near end crosstalk or NEXT:** When the “victim” port is on the same side as the “aggressor” port

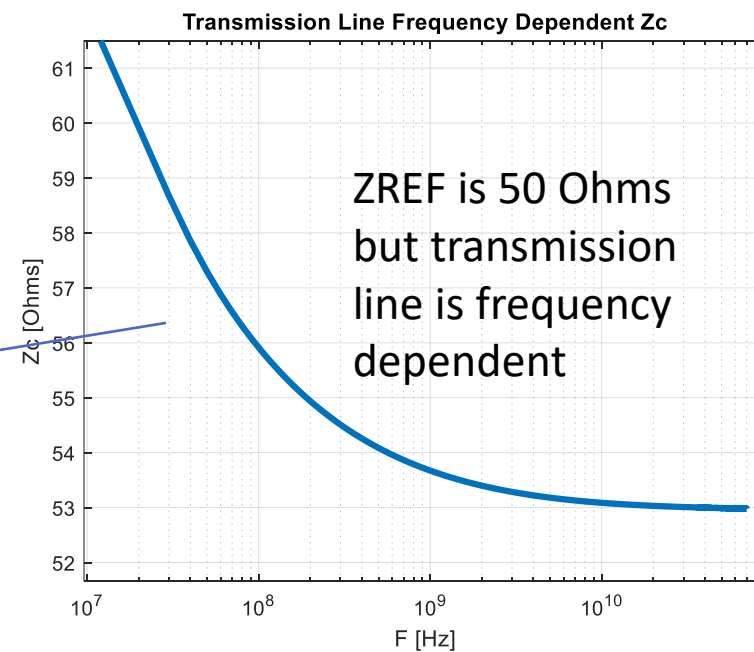
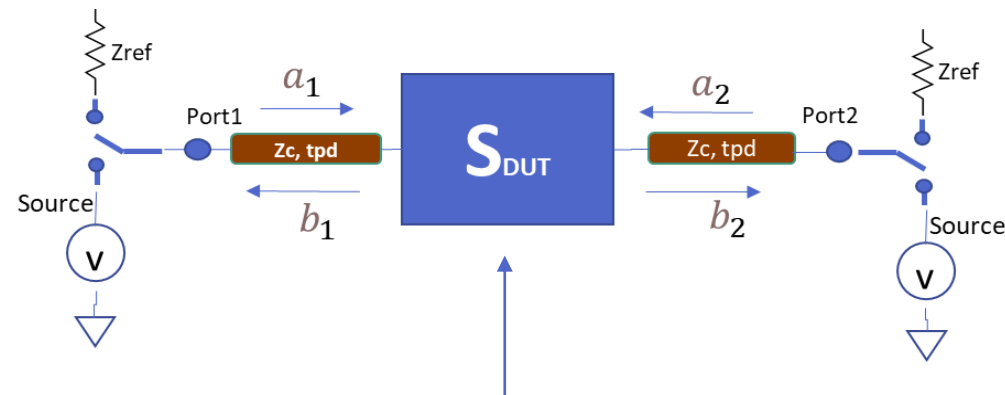
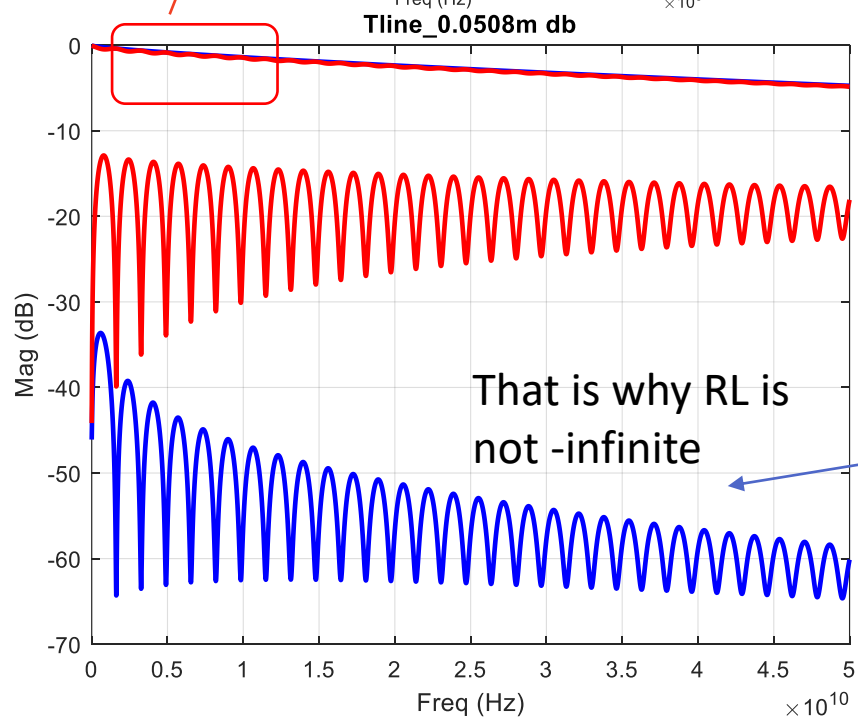
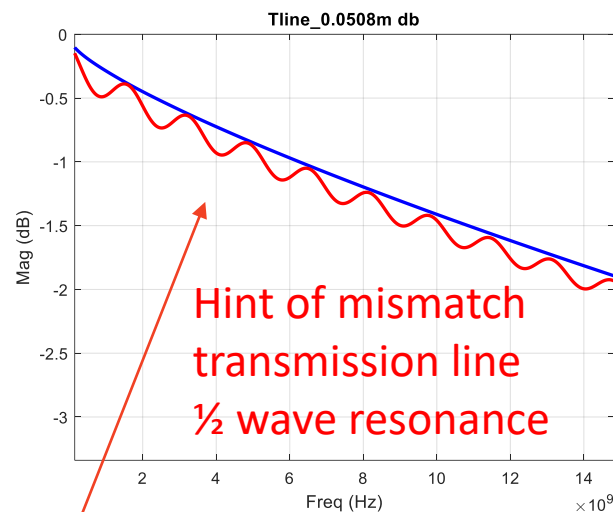


Resonances in S-parameters



- In time domain every point, represent a time and location (easy to understand)
- In frequency domain, every frequency point represents all locations at all times.

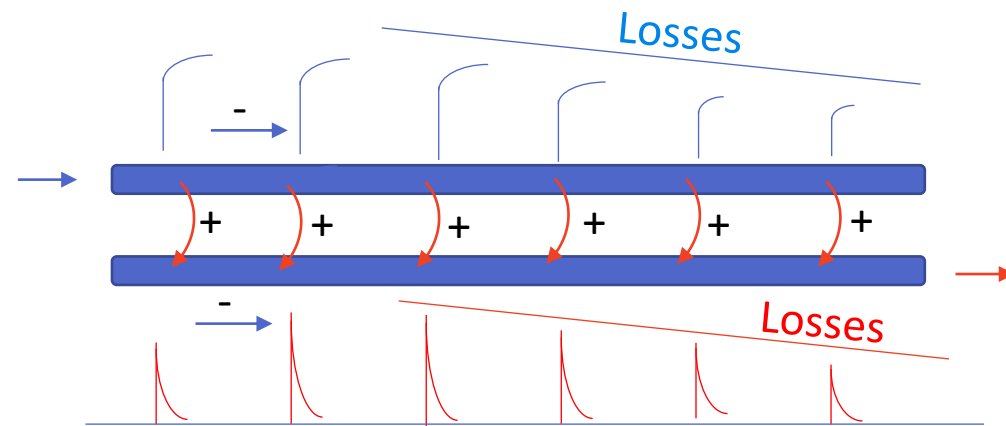
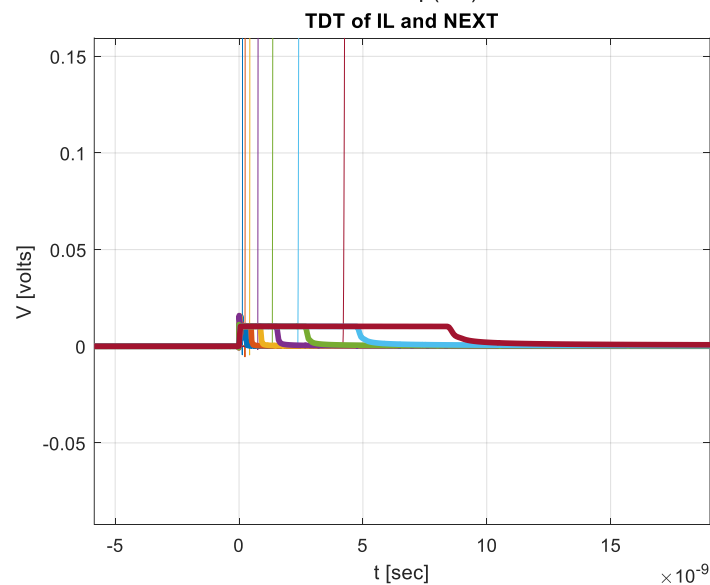
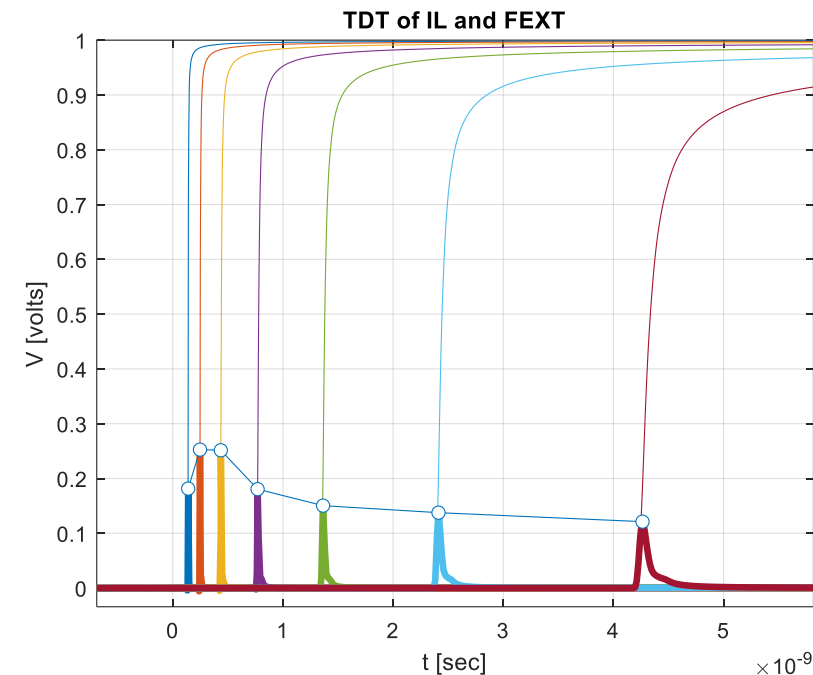
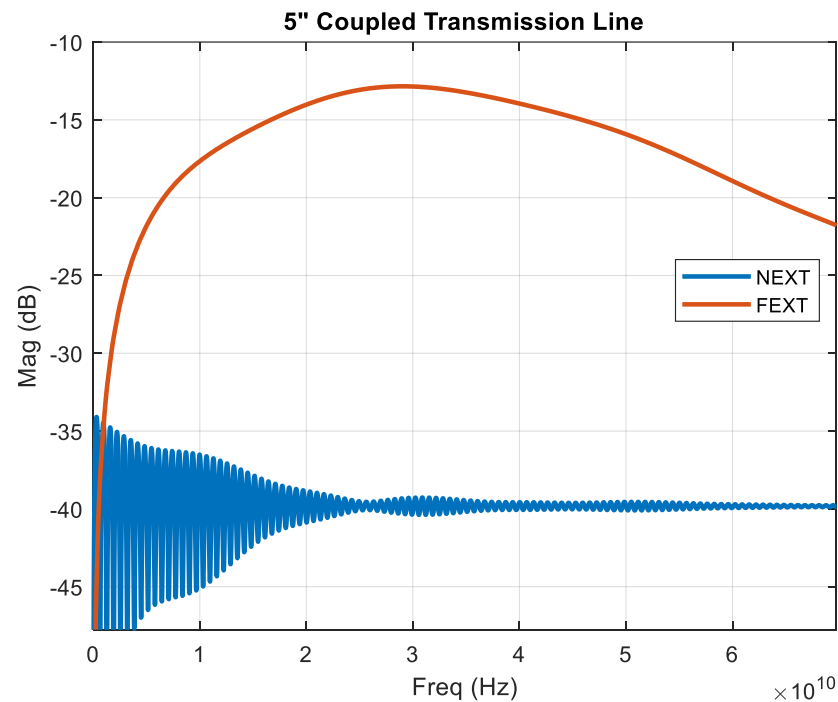
Transmission Line IL and RL



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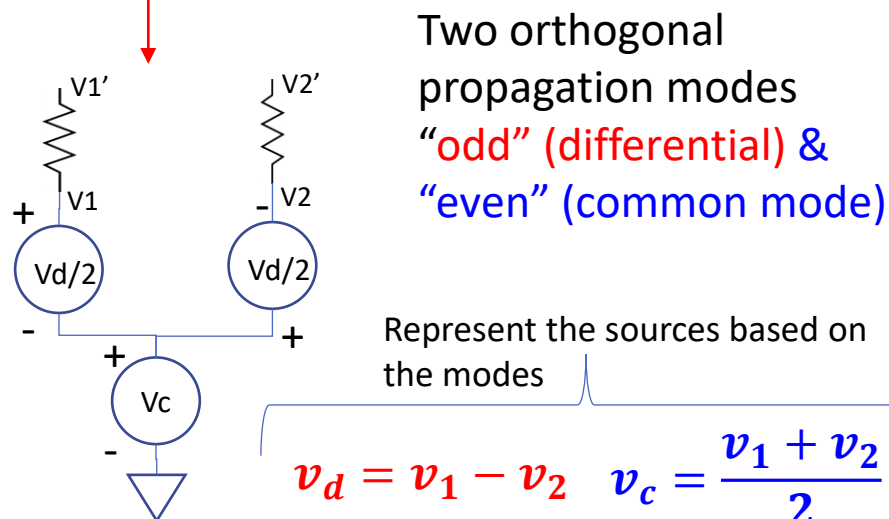
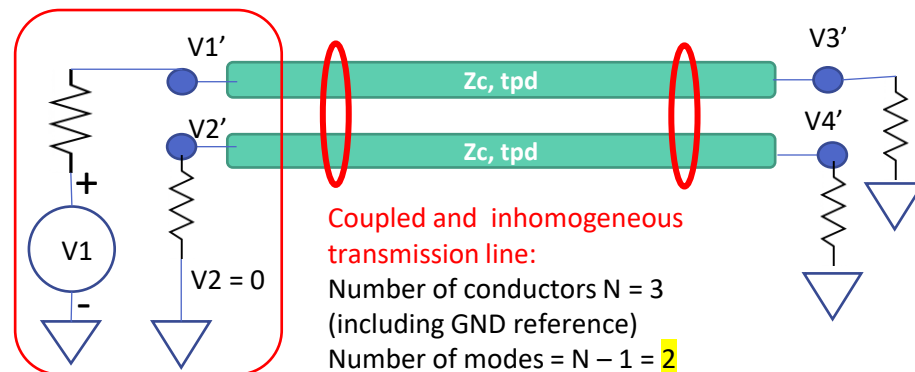
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FEXT and NEXT Crosstalk



Example, Single Ended In Differential Interconnect (Problem Introduction)

http://www.electrical-integrity.com/Paper_download_files/DC07_SUN_difflosses_v14.pdf



$$v_1 = v_c + \frac{v_d}{2} \quad v_2 = v_c - \frac{v_d}{2}$$

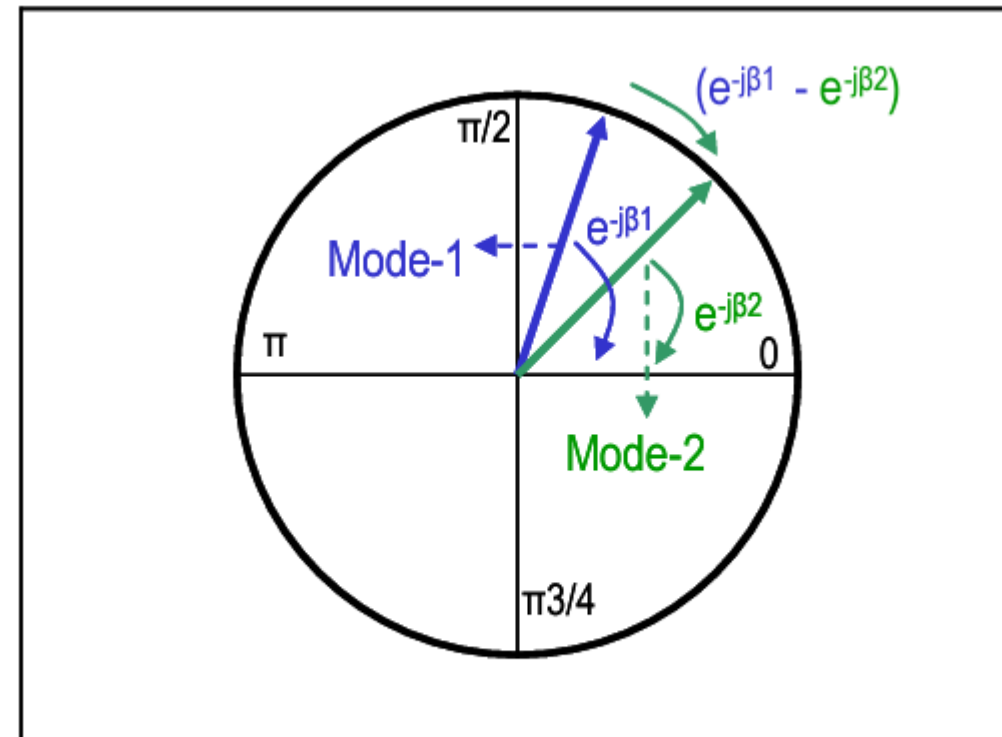
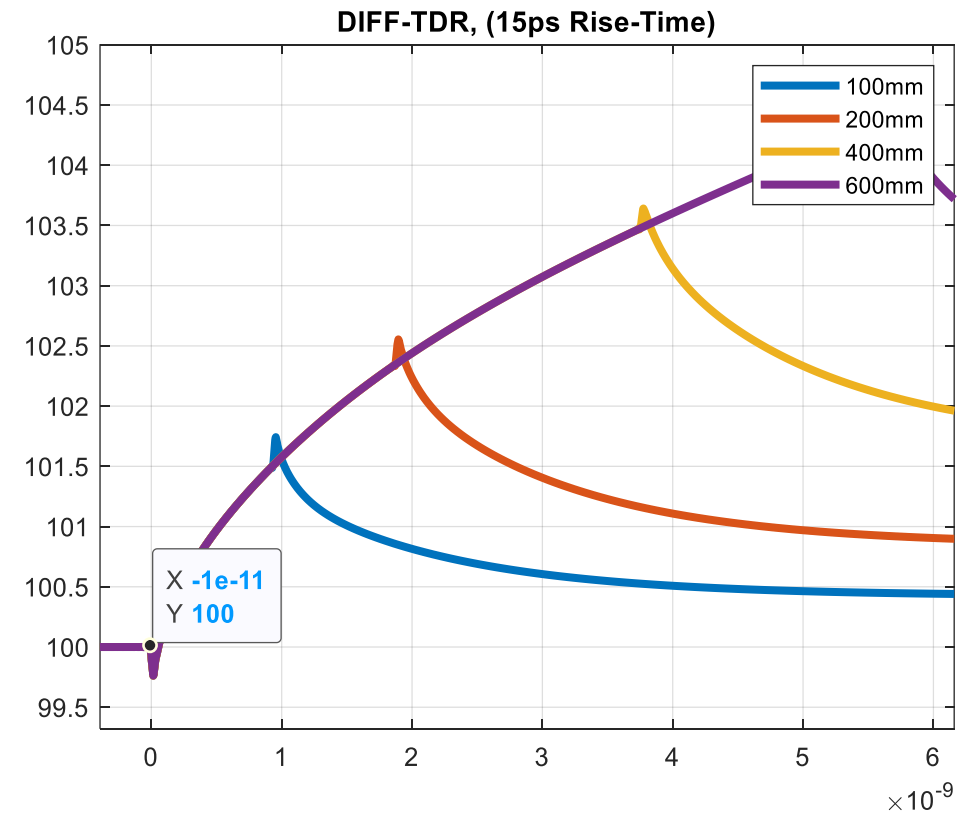
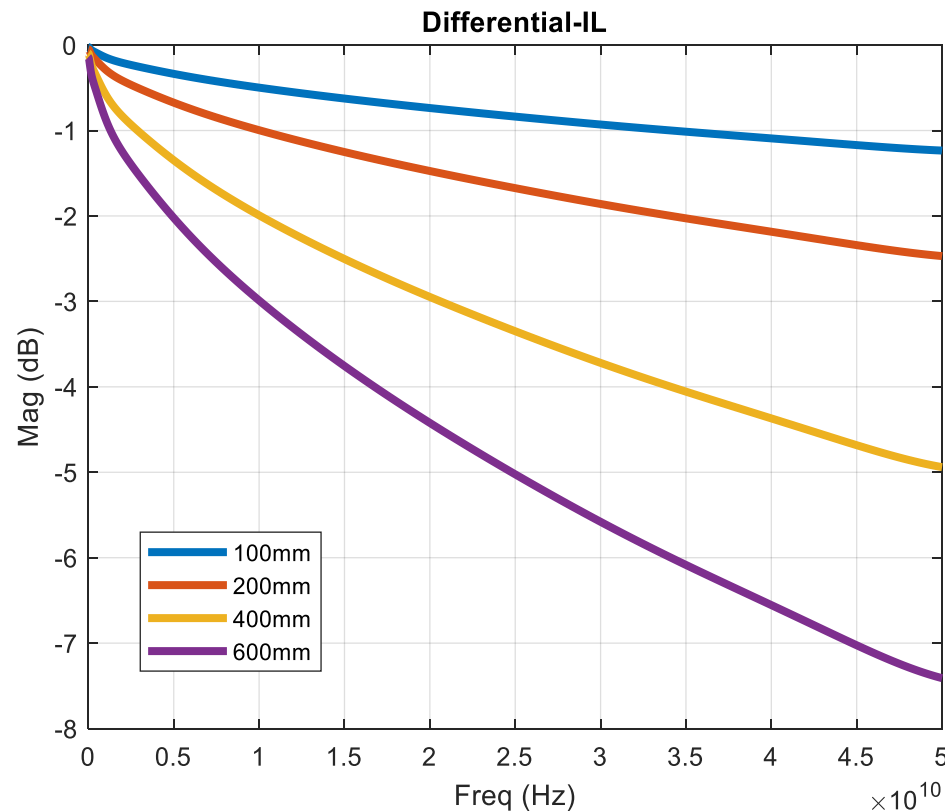


Figure 10: Phasor representation of the modal velocities.

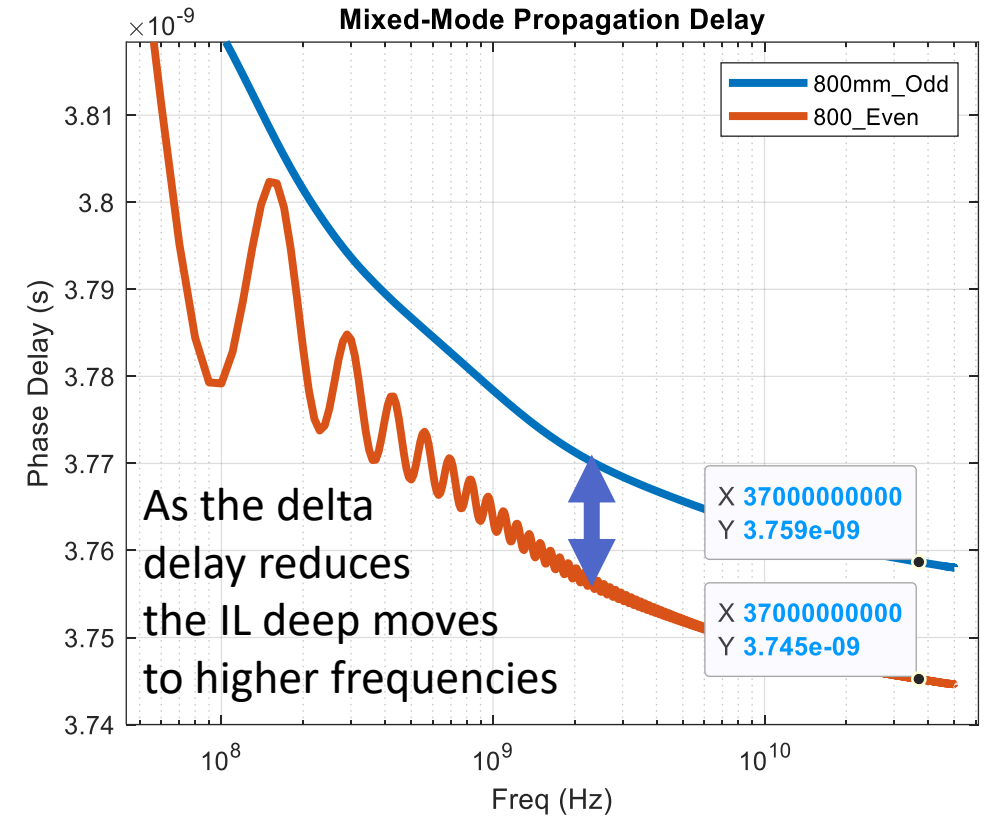
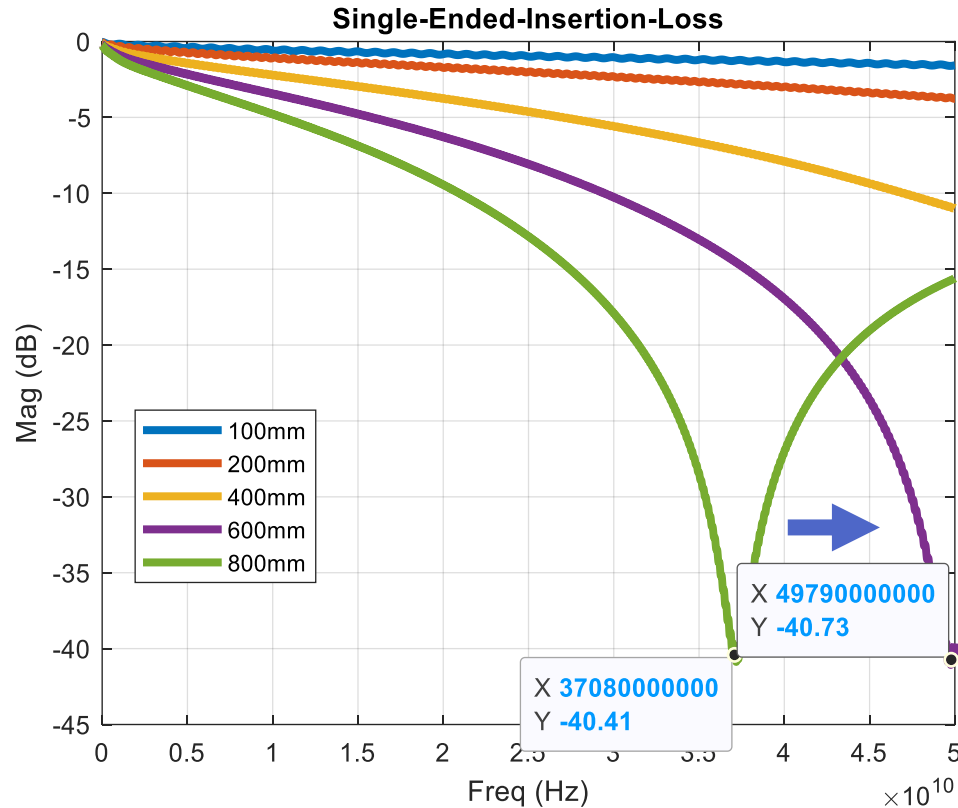
If the modes propagation delay are different, as we move through the line the vectors will constructively add or **subtract**. For a long enough line will see increased IL

Differential IL and TDR

- Very clean and uniform differential transmission.

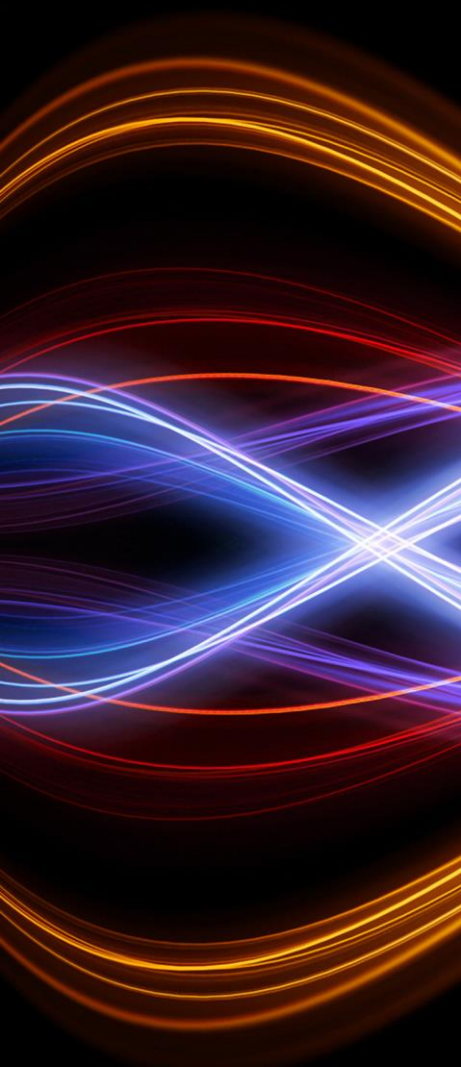


Single Ended Results in Differential System



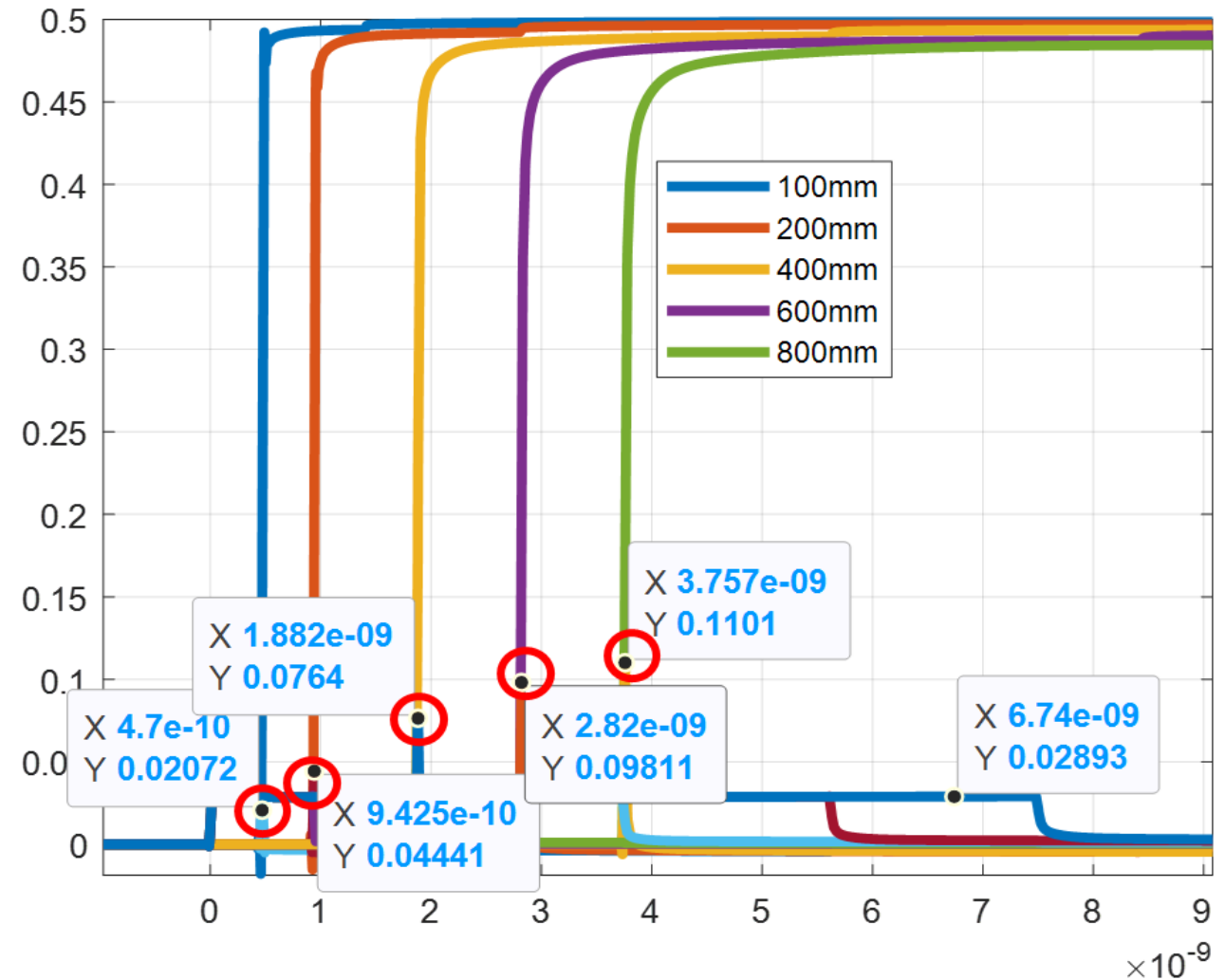
These are assuming one pair is terminated to 50 Ohms

- Resonances will be length dependent and related to $2/(\text{delta-modal delay})$
- Difference in odd and even propagation delay



Far End and Near End Crosstalk

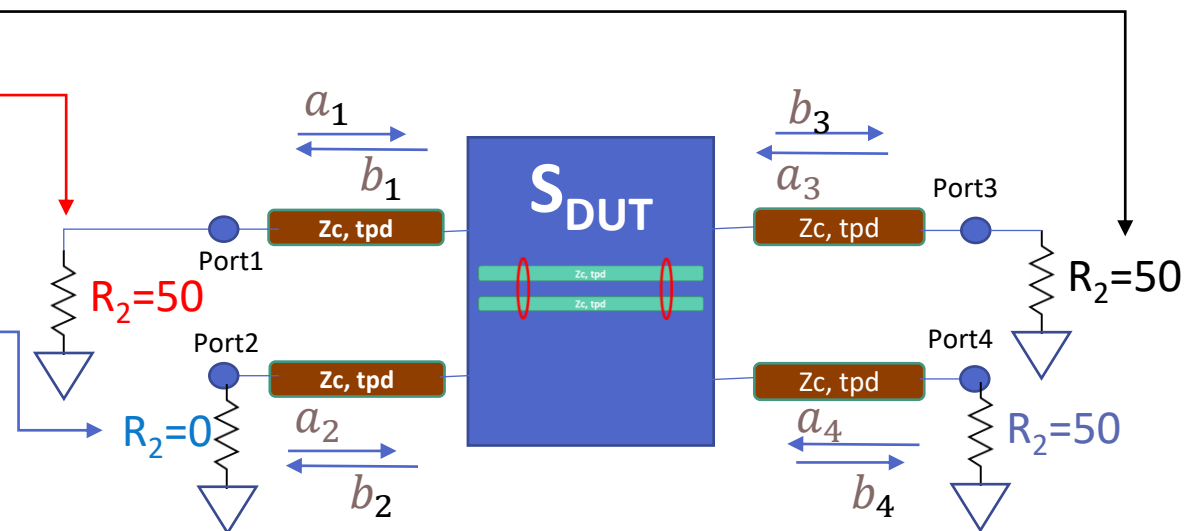
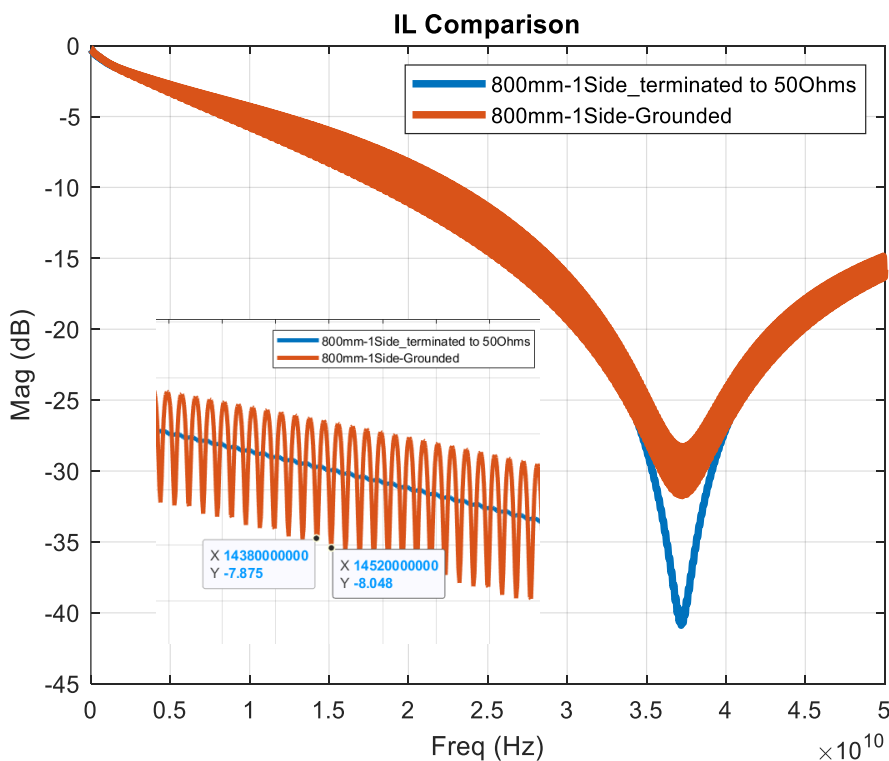
- Far end crosstalk indicating inhomogeneous medium.
- The longer the length the higher the FEXT accumulation, until due to losses will start to level and decay



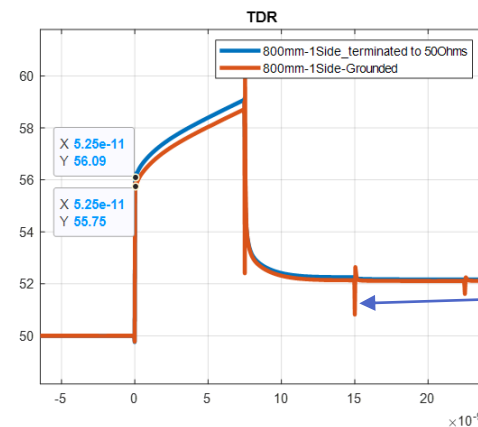
Grounding One Side of a Diff-Pair

Simply renormalizing
S-parameters
(no need of anything else)

$$S_{DUT_R} = \text{Renorm}(S_{DUT}, [50, 0, 50, 50])$$



- Grounding one side of a diff-pair produces resonances related to the length of the cable



Resonances in the
Time domain

Summary

- S-parameters introductions and evolution from Z and Y
- Mixed Mode S-parameters and generation for other cases
- Nomenclature
- S-parameter features normally found
 - Resonances
 - Crosstalk
 - Reference
- Single Ended in Differential Pair example



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