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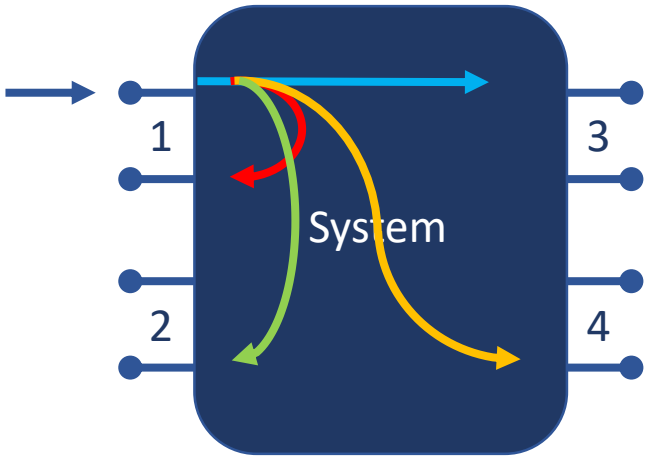
Causality correction or no causality correction? That is the question.

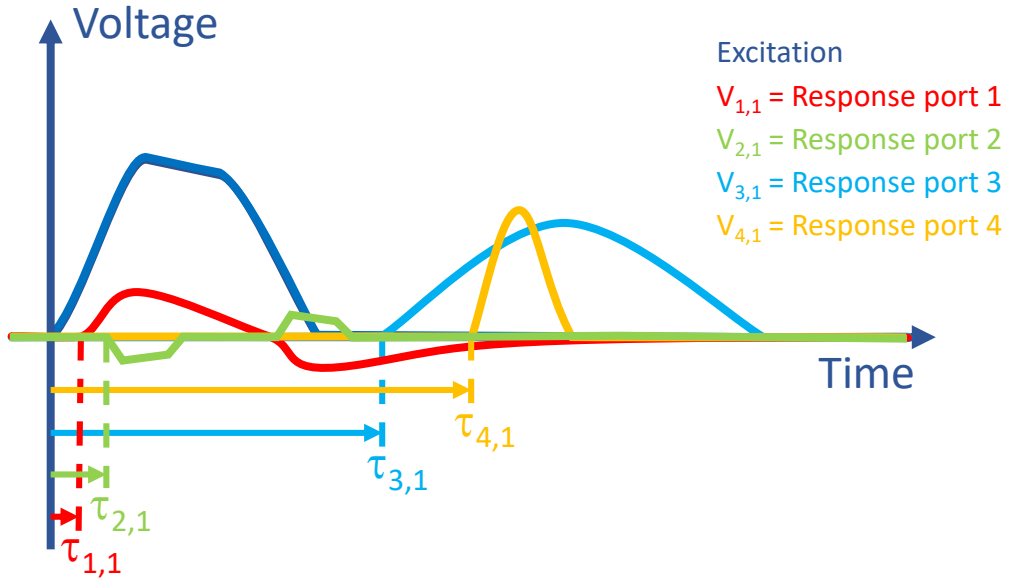
Stefaan Sercu

Introduction

- What is causality?
- Examples of non causality
- Causality correction methods and the impact on the SI performance
  - Causal S-parameter models
  - Accurate, causal time domain response
- Conclusions

What is Causality?

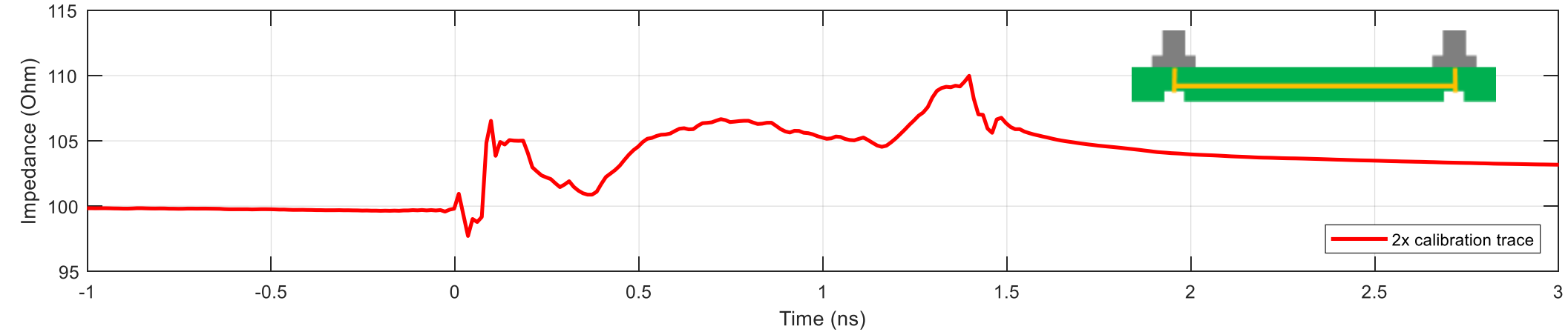


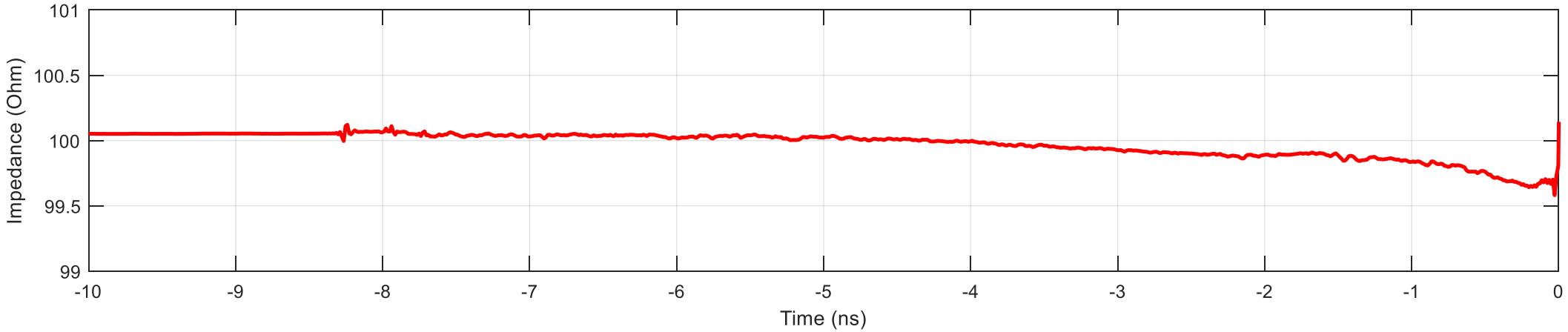


System is causal $\Longleftrightarrow$ $\tau_{i,j} \geq 0$ $\Longleftrightarrow$ $V_{i,j}(t \leq \tau_{i,j}) = 0$

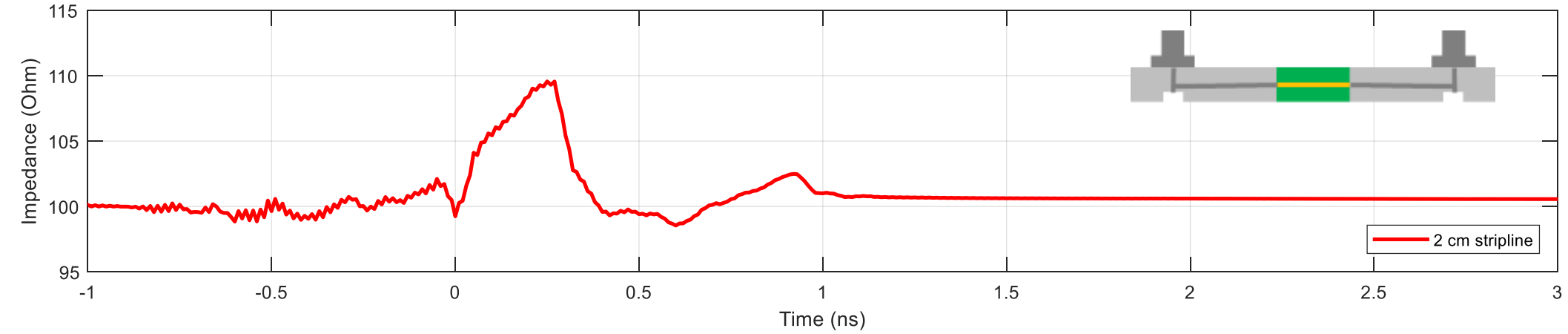
$\tau_{i,j}$ = system response time at port i for an excitation at port j

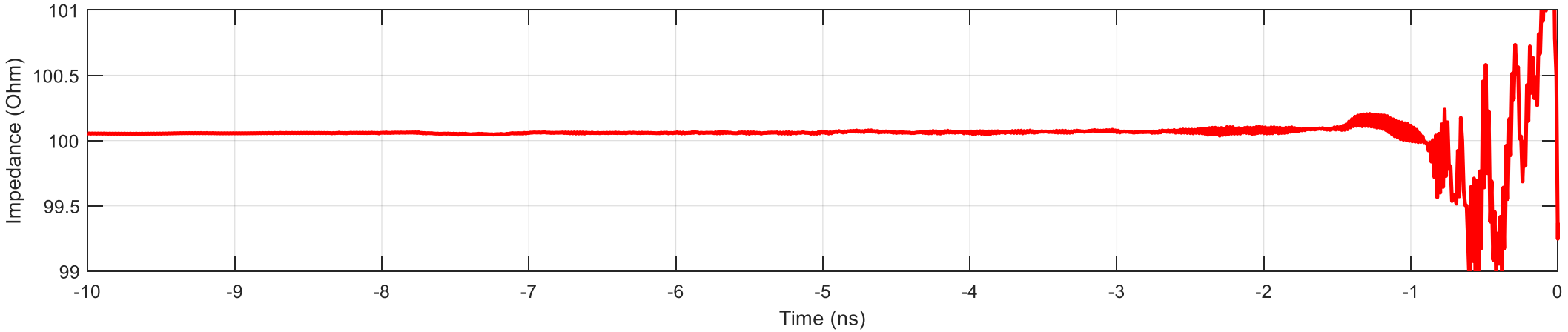
Examples of Non-Causality Calibration





Examples of Non-Causality Test Fixture De-embedding

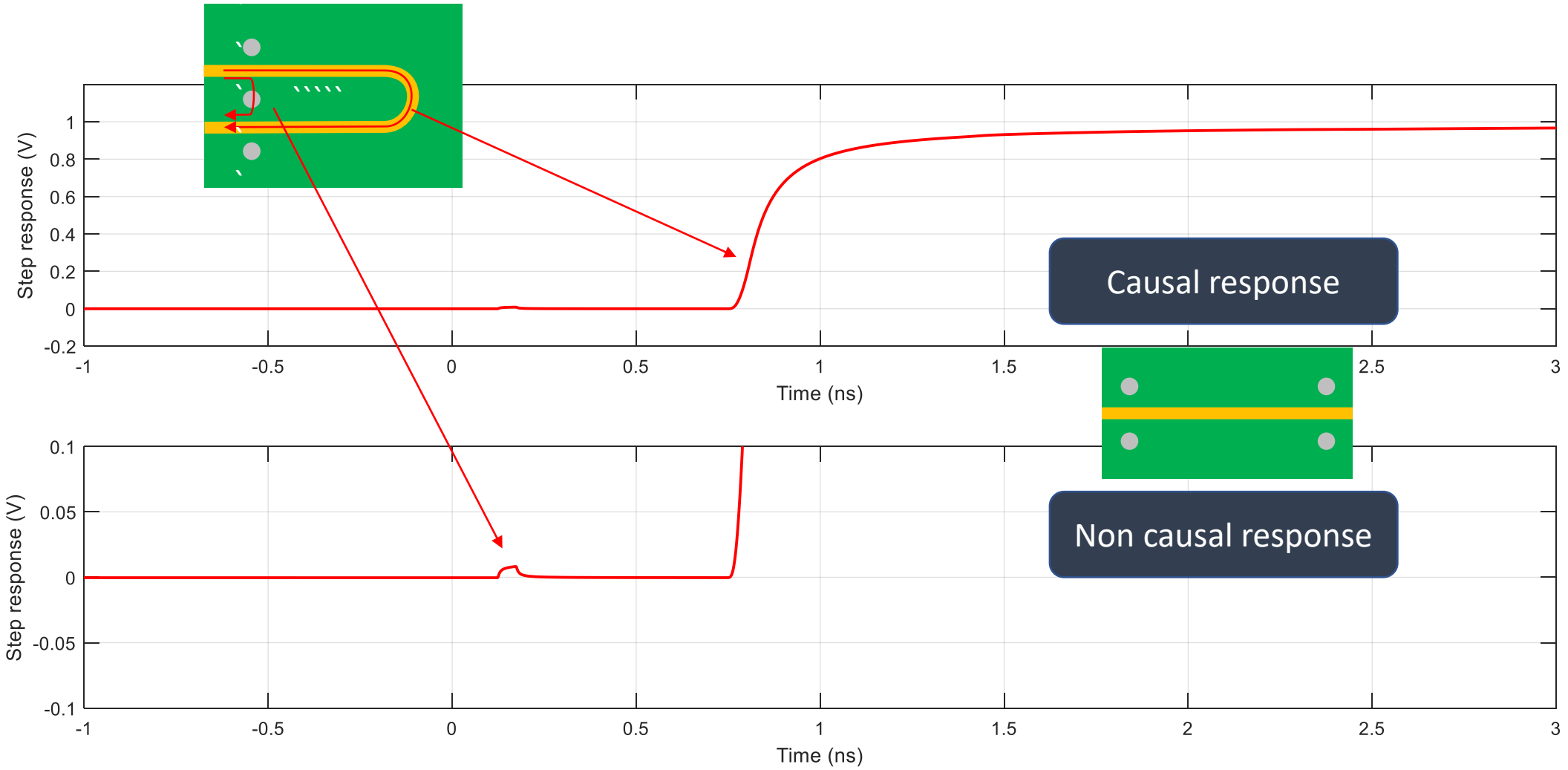




Examples of Non-Causality
Full wave modeling

TO BE INCLUDED

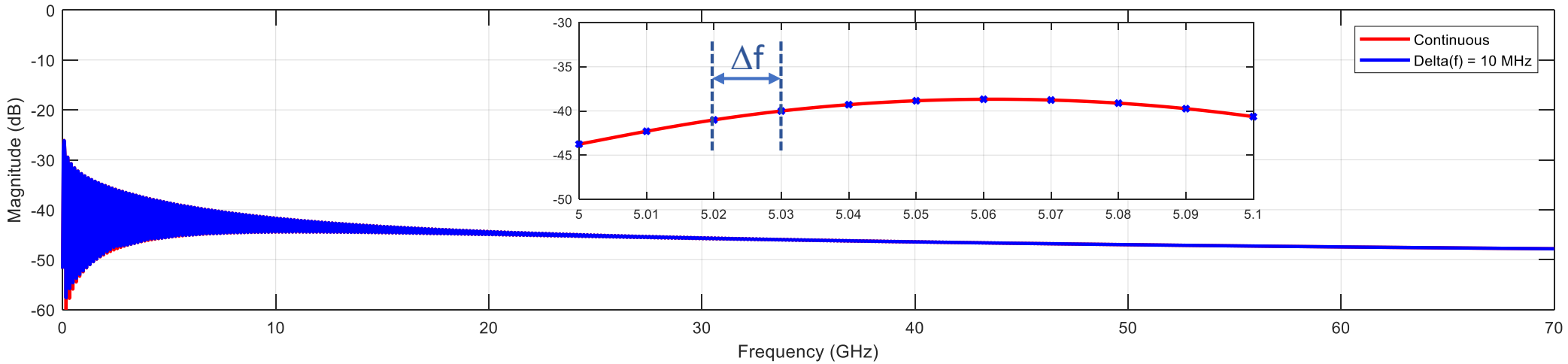
Examples of Non-Causality Modeling

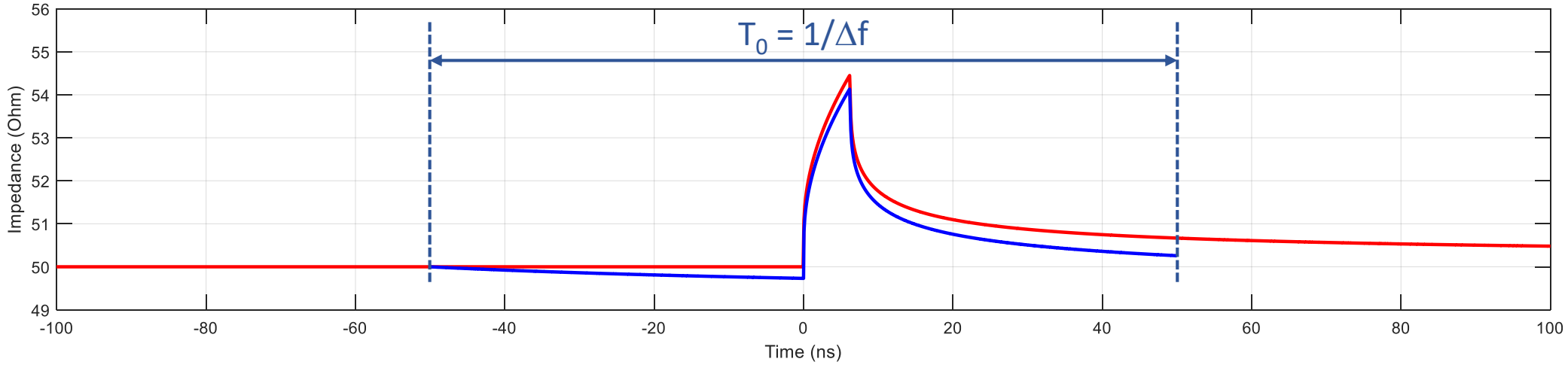


Classification of Non-Causalities

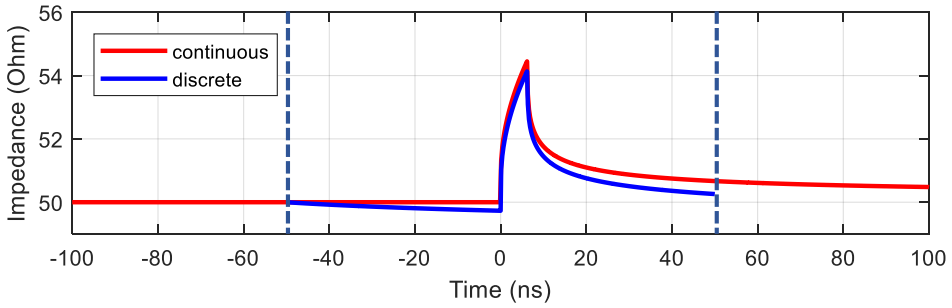
- Physical or Real Non-Causalities
  - S-parameter Measurements
    - Calibration, test-fixture de-embedding, equipment imperfections
  - S-parameter Modeling
    - Full wave models
    - Equation based models
      - Material properties, surface roughness model
- Numerical Non-Causalities
  - Discretization
  - Bandwidth limitation

Discretization of S-parameters





Discretization of S-parameters



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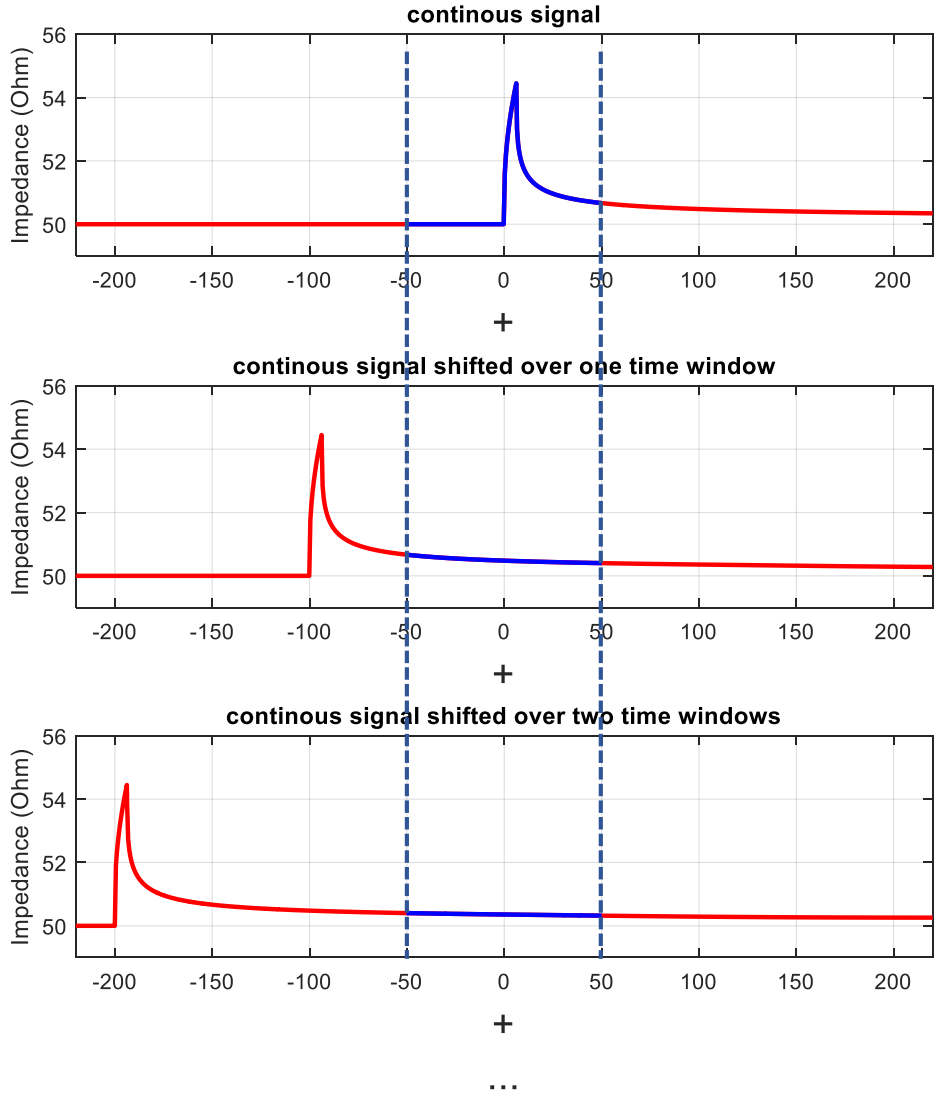


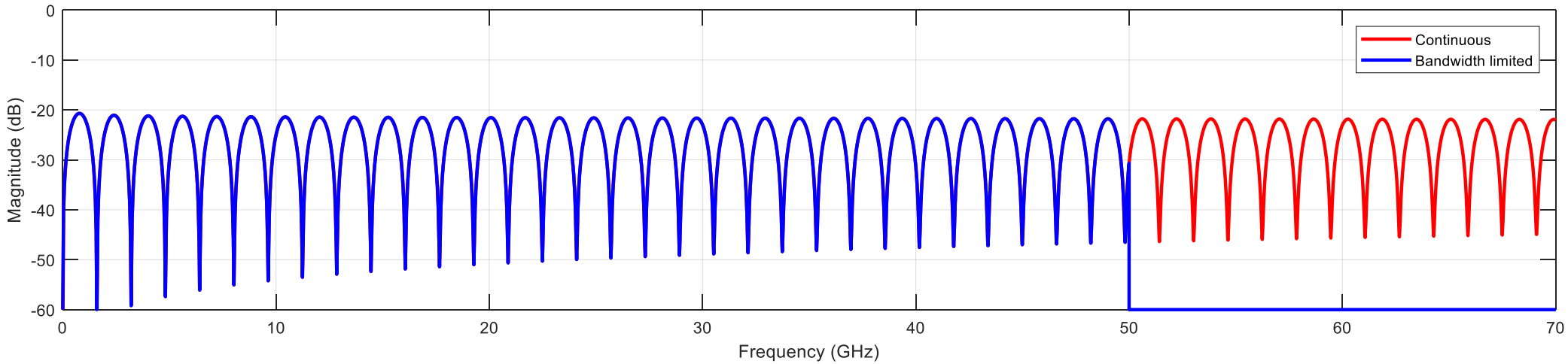
What causes the Non-Causality?

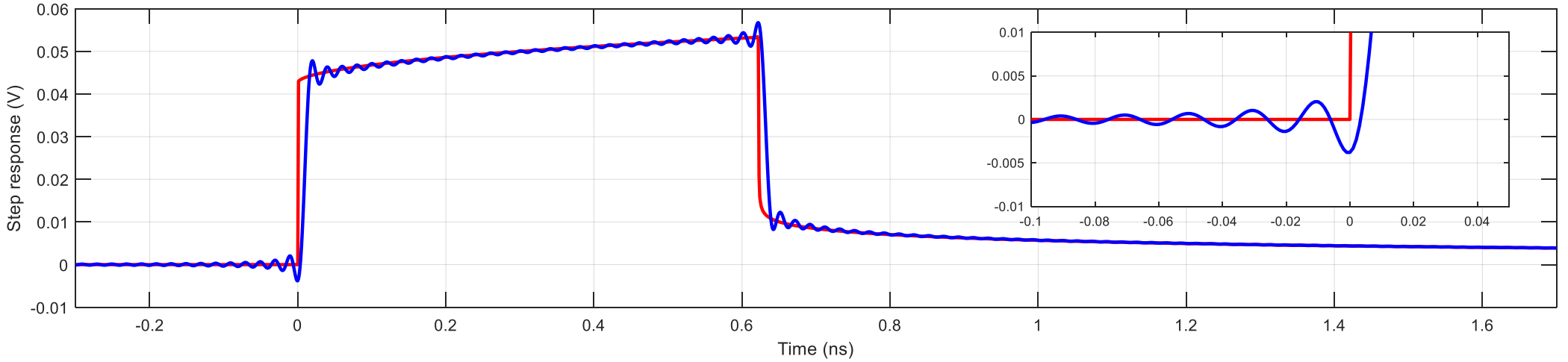
- System time response does not fall completely within the time window.

How to reduce the Non-Causality?

- Non-Causality can be reduced by increasing the time window, thus by reducing the frequency step so that the complete response falls in the time domain window.

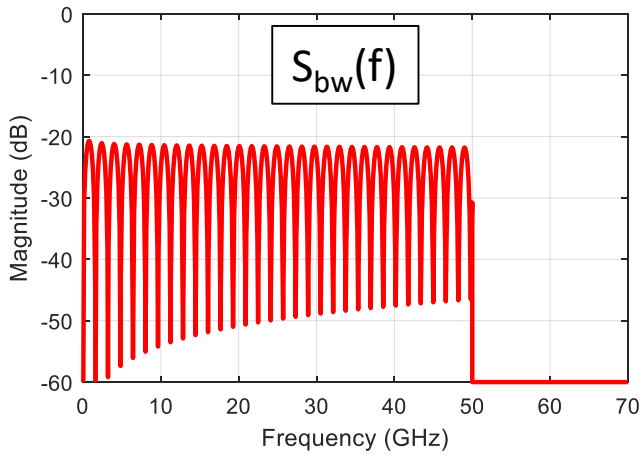
Bandwidth Limitation of S-parameters





Bandwidth Limitation of S-parameters

$$S_{bw}(f) = S_{inf}(f).Block_{50}(f)$$



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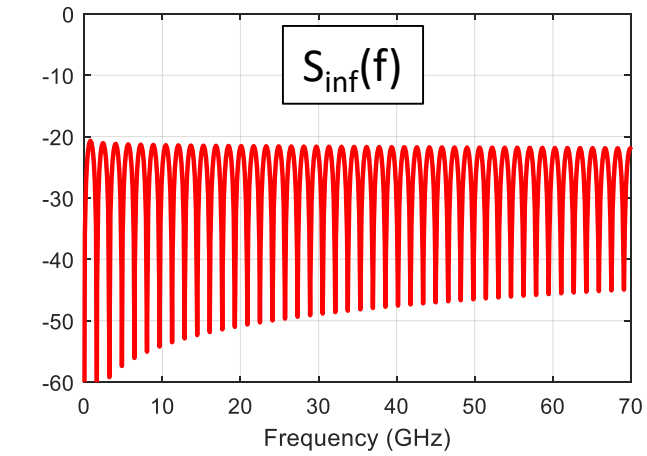

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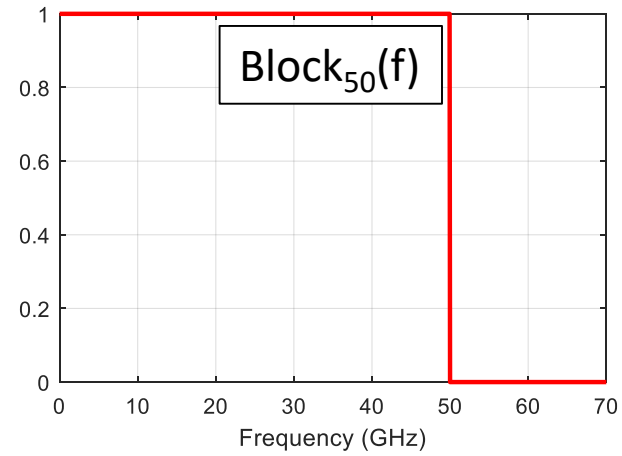
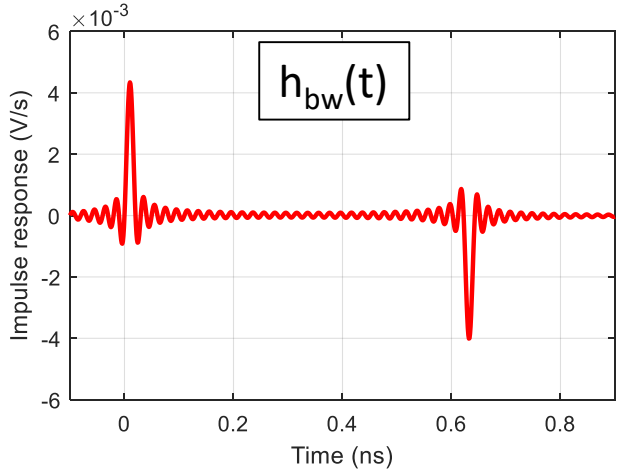


Inverse Fourier Transform

$$h_{bw}(t) = h_{inf}(t) * sinc(t) = \int_{-\infty}^{+\infty} h_{inf}(\tau).sinc(t-\tau).d\tau$$



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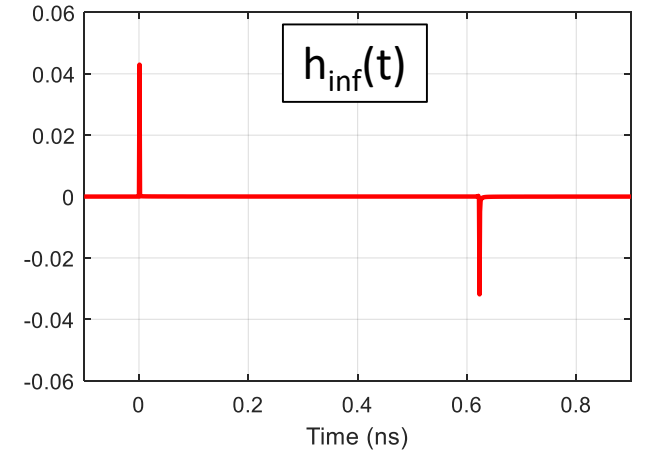

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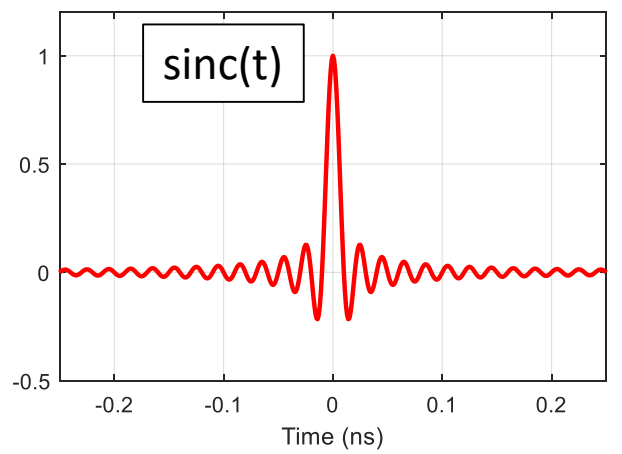


Causality Correction Methods

- Purpose of the corrected S-parameter model
  - Use model to build a complete channel
    - Correct physical non-causalities
    - Correcting numerical non-causalities leads to accuracy loss
  - Calculate time domain metrics to take decisions
    - Correct physical non-causalities
    - Time domain results should be as if bandwidth is infinite and frequency step is continuous.
  - Calculate time domain metrics which are not used for decisions
    - Don’t care
- Remark
  - Causality correction methods should be avoided: find the root cause and correct it

Causality Correction Methods
Hilbert Transform

$S(f) = S_R(f) + i\,S_I(f)$ is causal $\Leftrightarrow$

$$S_R(f) = \frac{1}{\pi} P \int_{-\infty}^{+\infty} \frac{S_I(f')}{f - f'} df'$$

$$S_I(f) = -\frac{1}{\pi} P \int_{-\infty}^{+\infty} \frac{S_R(f')}{f - f'} df'$$

$h(t) = h_{even}(t) + h_{odd}(t)$ is causal

$h(t) = 0 \quad t<\tau$ $\Leftrightarrow$

$$v_{even}(t) = sign(t-\tau).v_{odd}(t)$$

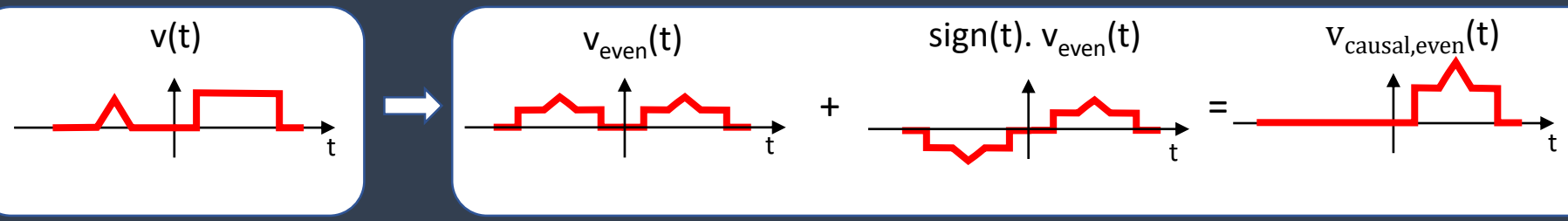
$$v_{odd}(t) = sign(t-\tau).v_{even}(t)$$

$$sign(t) = \begin{cases} -1 & t < 0 \\ 0 & t = 0 \\ 1 & t > 0 \end{cases}$$

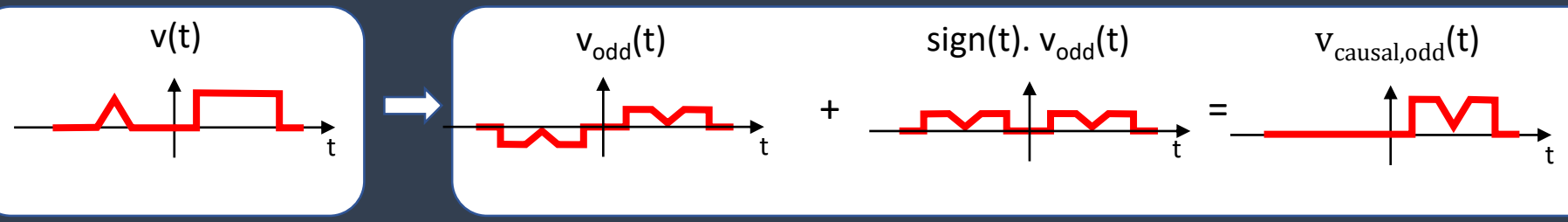
REAL and IMAGINARY PART or ODD and EVEN part of a causal signal are linked through the HILBERT TRANSFORM

Causality Correction Methods
Hilbert – real - image

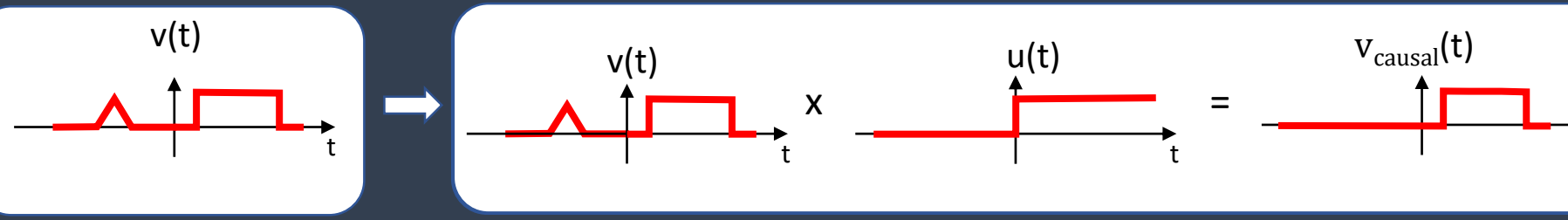
$V_{causal,even}(t) = v_{even}(t) + sign(t).\ v_{even}(t)$



$V_{causal,odd}(t) = v_{odd}(t) + sign(t).\ v_{odd}(t)$

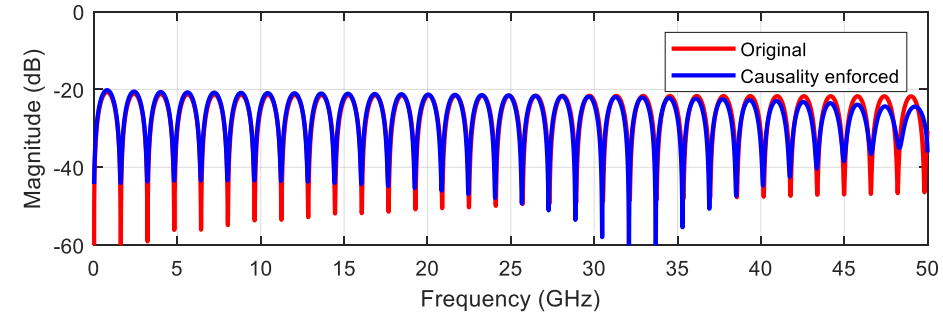


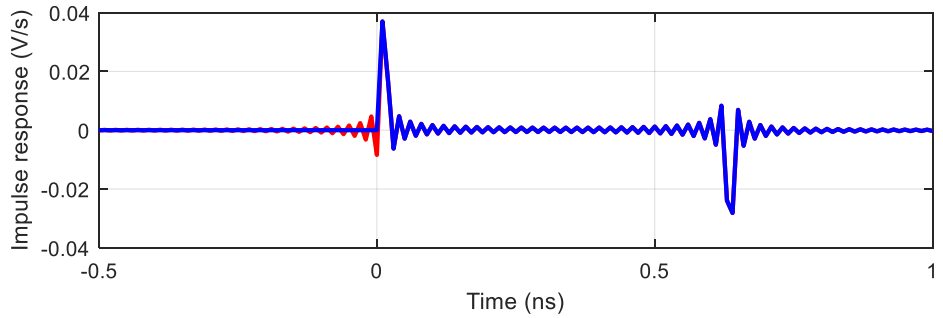
$V_{causal}(t) = u(t).v(t)$ - u(t) = perfect step signal

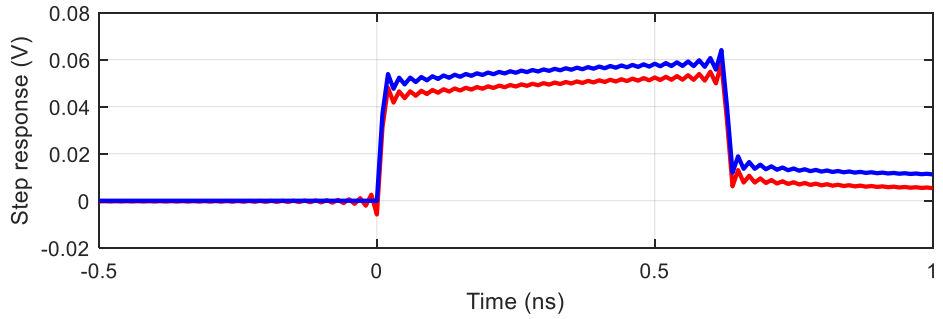


Causality Correction Methods
Hilbert – real - image

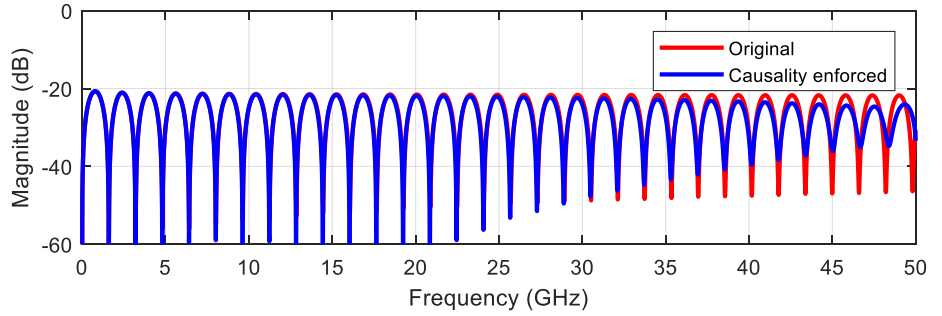
Impulse response enforcement

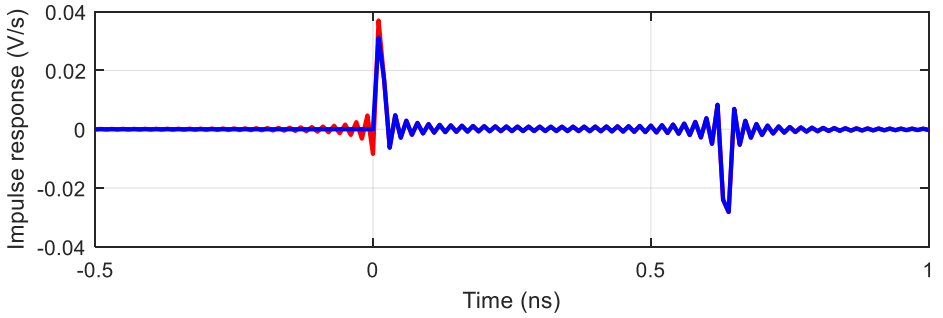


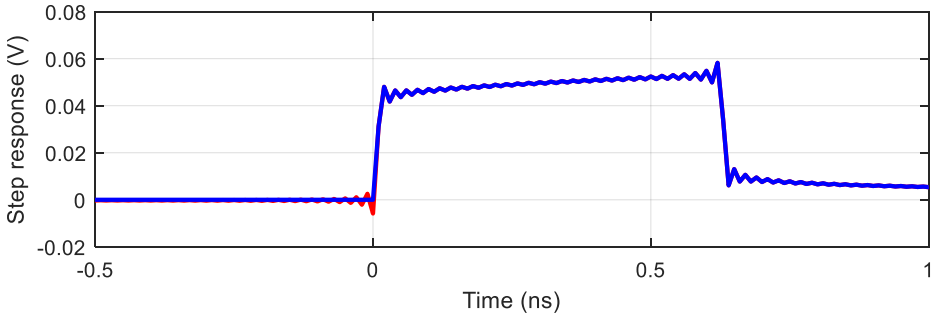




Step response enforcement







Causality correction methods
Hilbert – real - image

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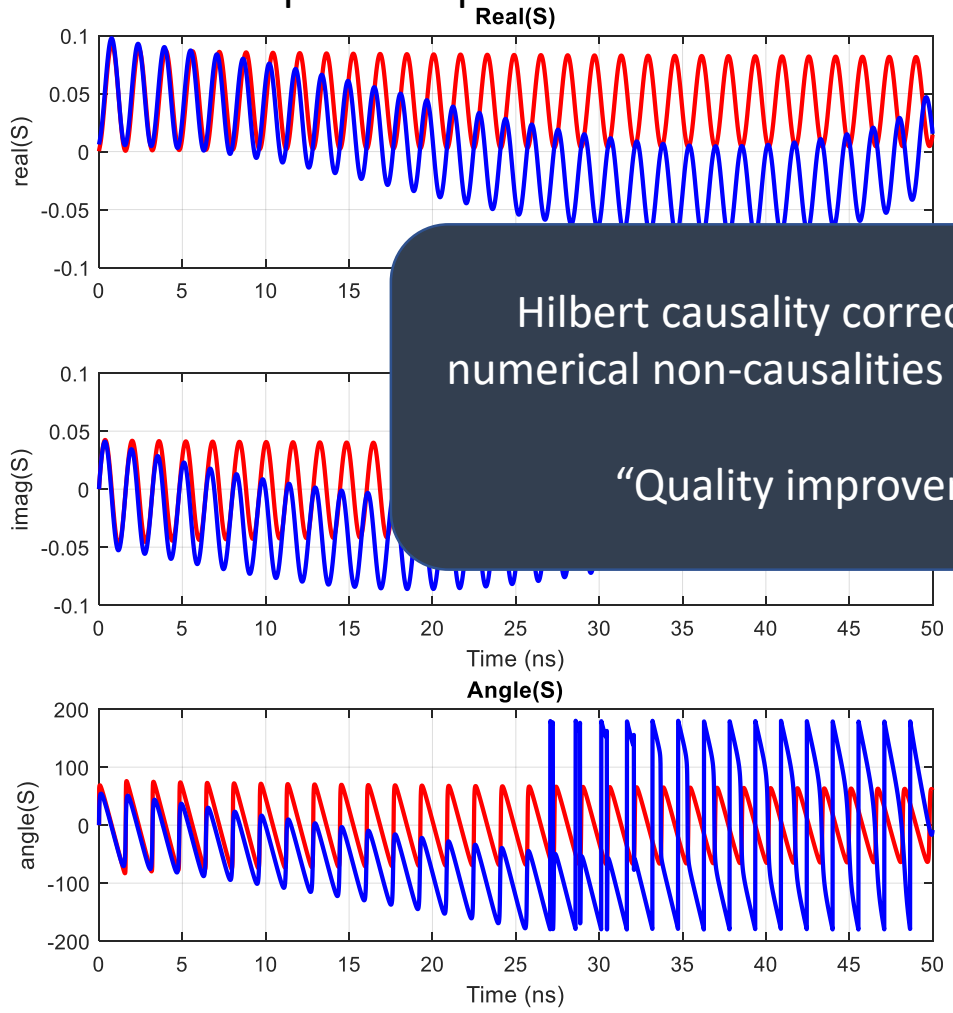

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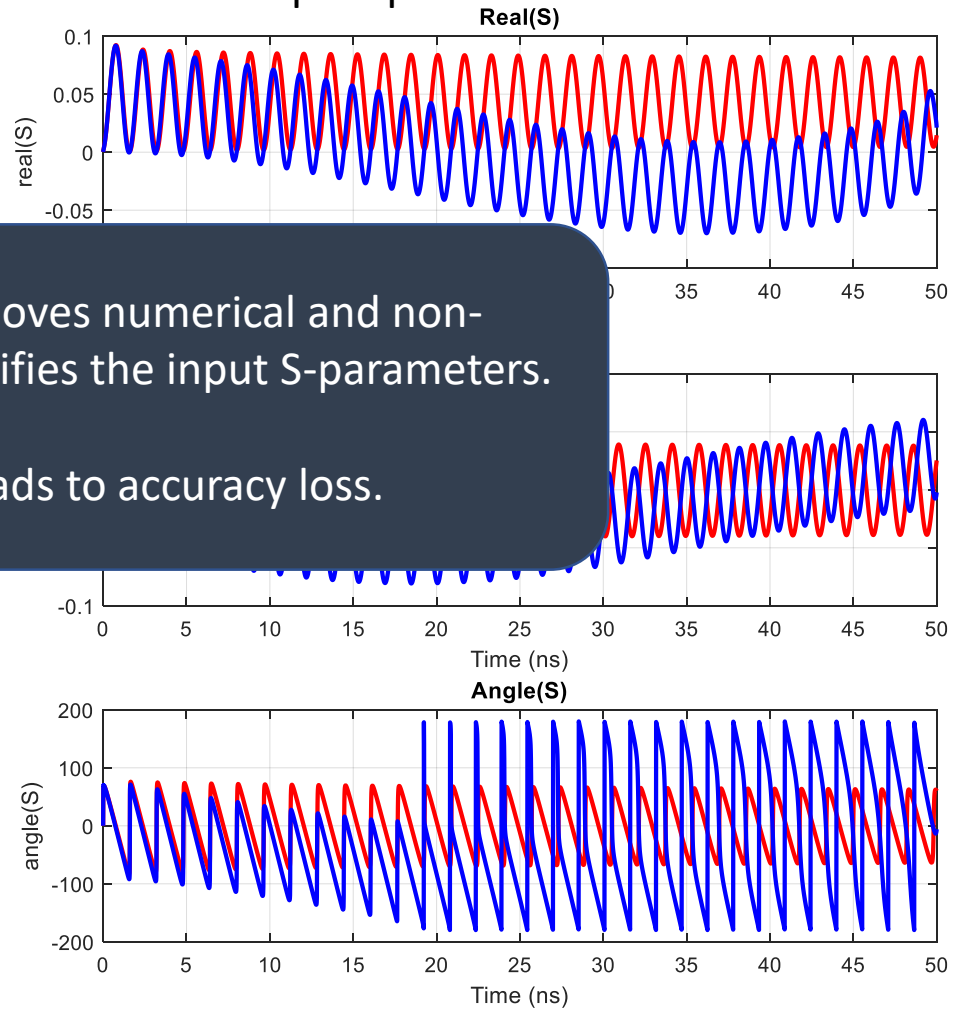


Hilbert causality correction removes numerical and non-numerical non-causalities and modifies the input S-parameters.

"Quality improvement" leads to accuracy loss.

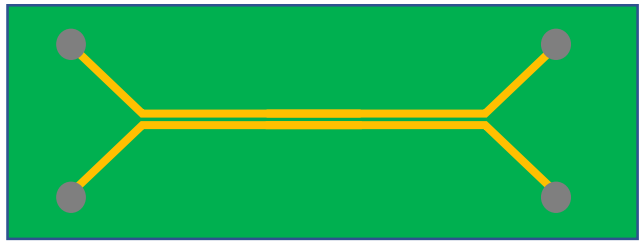
Causality Correction Methods
Hilbert – amplitude - phase

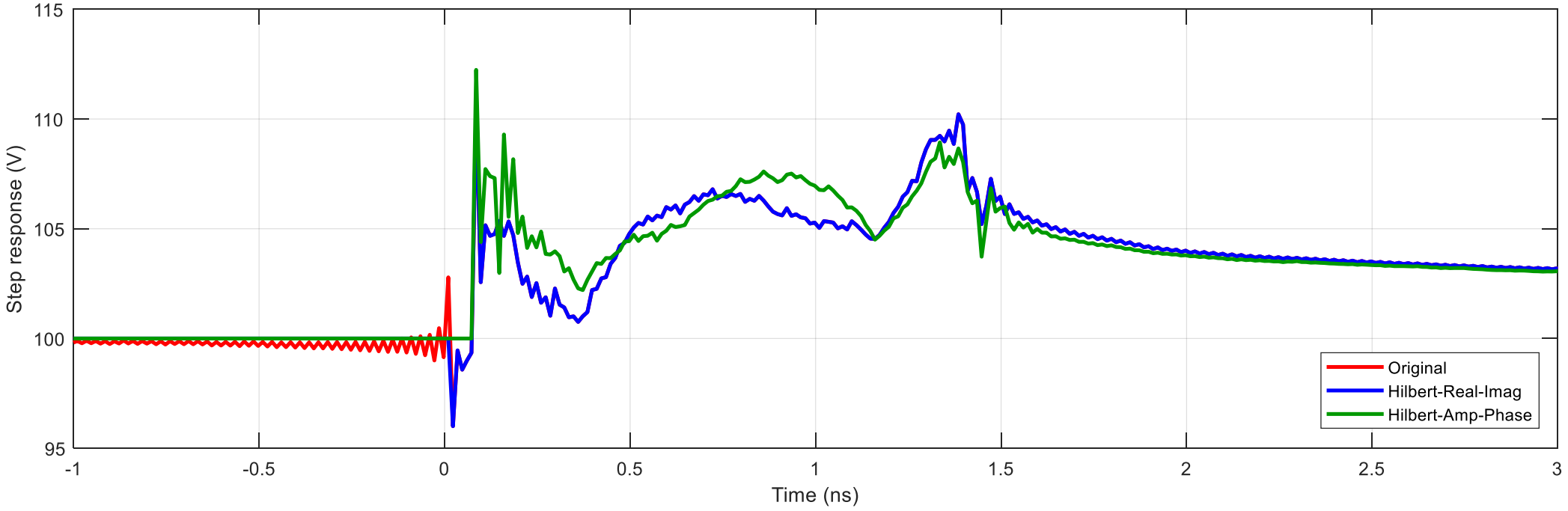
- Hilbert transform

$S(f) = A(f).e^{-i\varphi(f)}$ is causal $\Leftrightarrow$ $\log(S(f)) = \log(A(f))-i\varphi(f)$ is causal

$\log(A(f))$ and $-\varphi(f)$ are linked through the HILBERT TRANSFORM

Causality Correction Methods
Hilbert



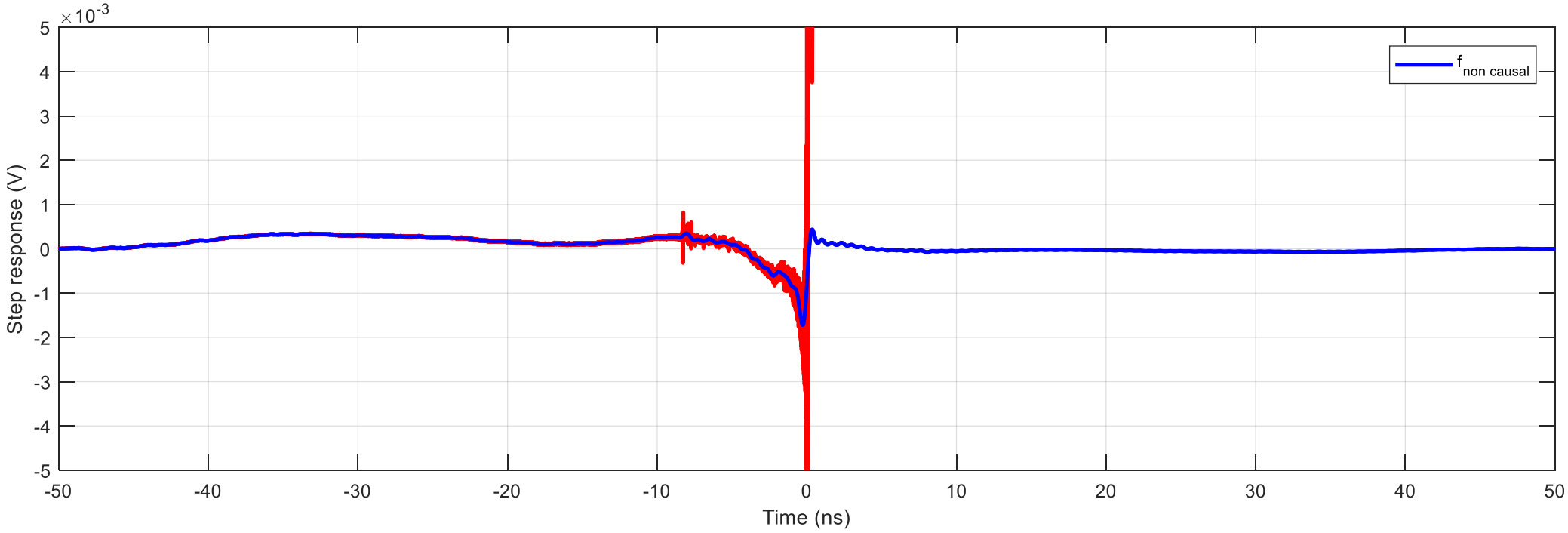


Causality Correction Methods

Correction without removal of numerical non-causalities

$$f(t_i) = A(f_0) + \sum_{n=1}^{N} A(f_n)\cos(n2\pi\Delta f t_i - \varphi(fn)) = R(f_0) + \sum_{n=1}^{N} R(f_n)\cos(n2\pi\Delta f t_i) + I(f_n)\sin(n2\pi\Delta f t_i)$$

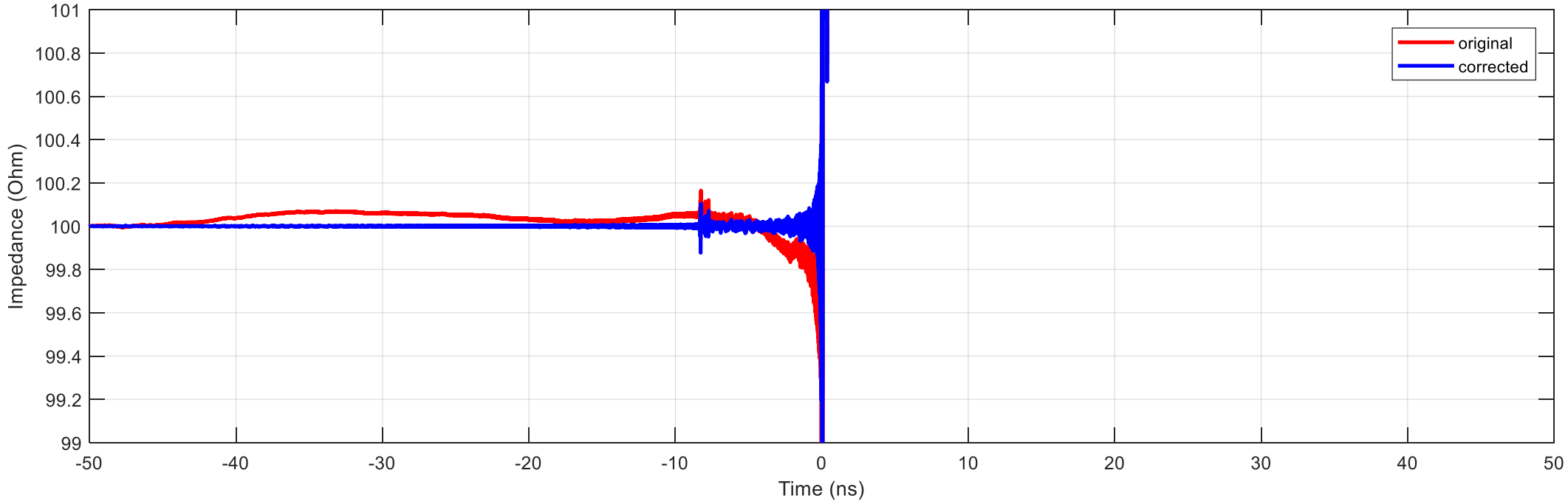
$$f_{non\ causal}(t_i) = R'(f_0) + \sum_{n=1}^{N_1} R'(f_n)\cos(n2\pi\Delta f t_i) + I'(f_n)\sin(n2\pi\Delta f t_i)$$



Causality Correction Methods

Correction without removal of numerical non-causalities

$$f_{corrected}(t_i) = f(t_i) - f_{non\ causal}(t_i)$$



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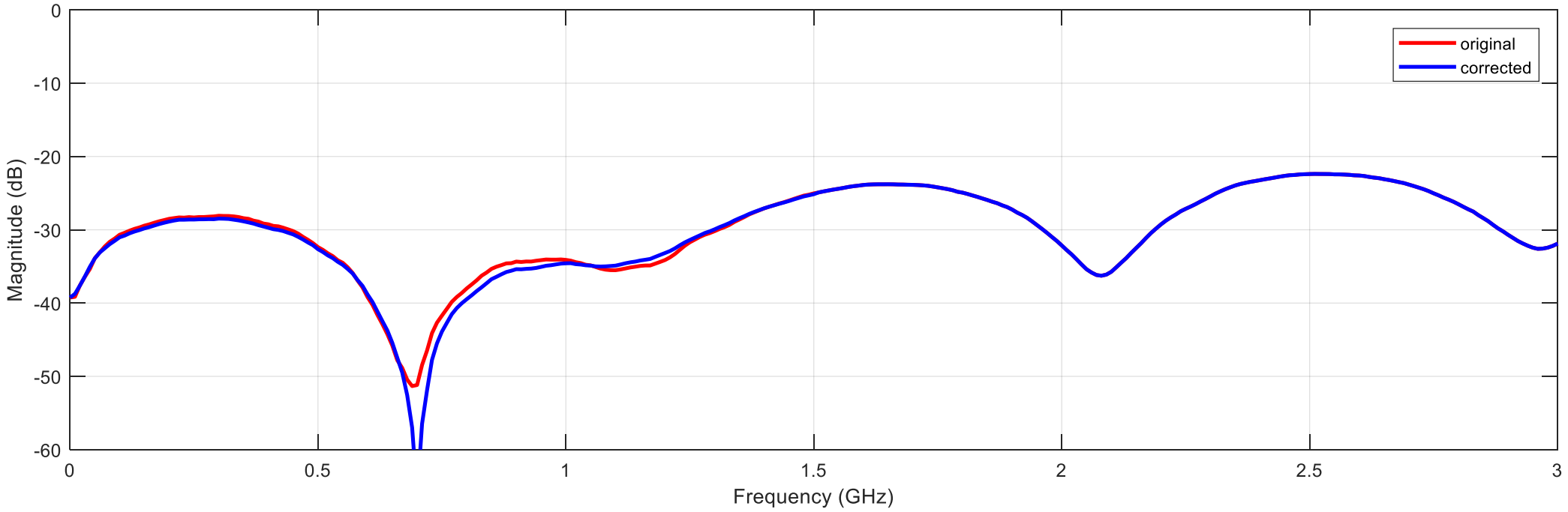
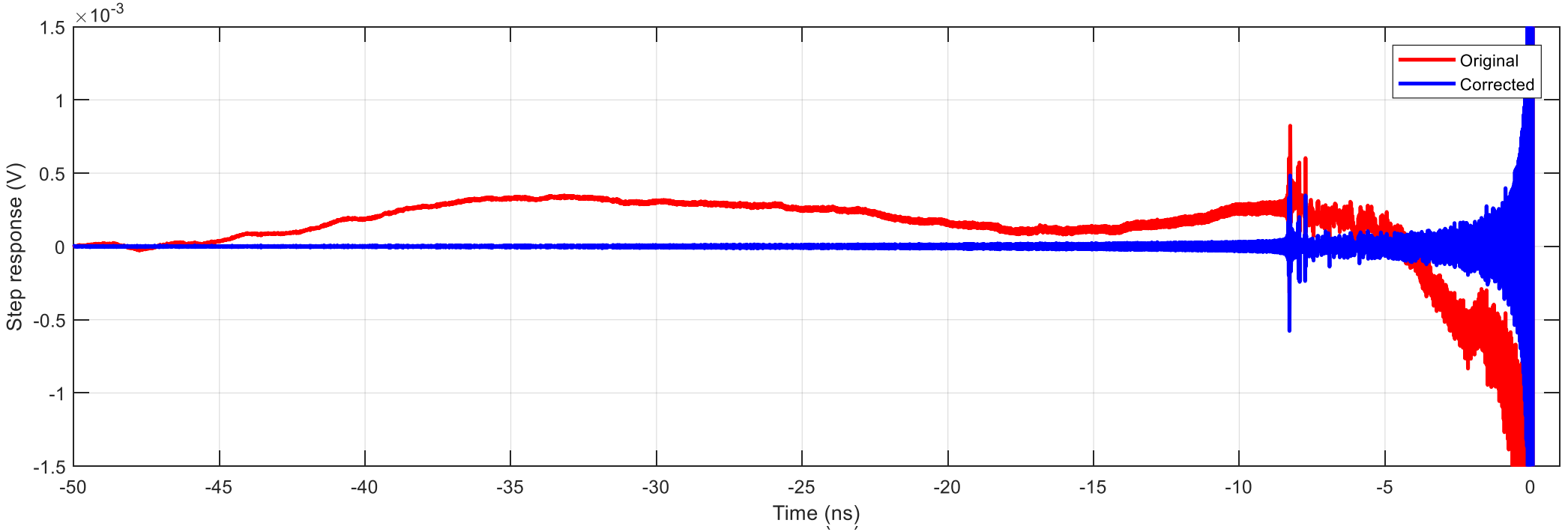


Causality Correction Methods

Correction without removal of numerical non-causalities

- Smooth signal at negative times
- Subtract from original signal



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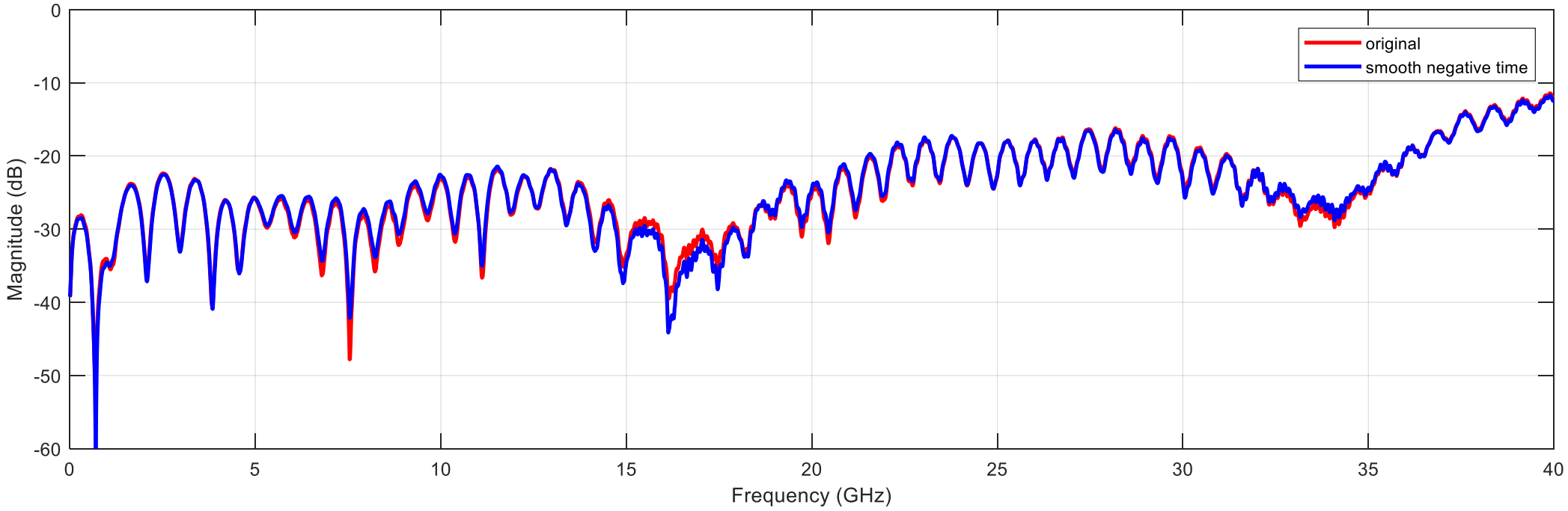
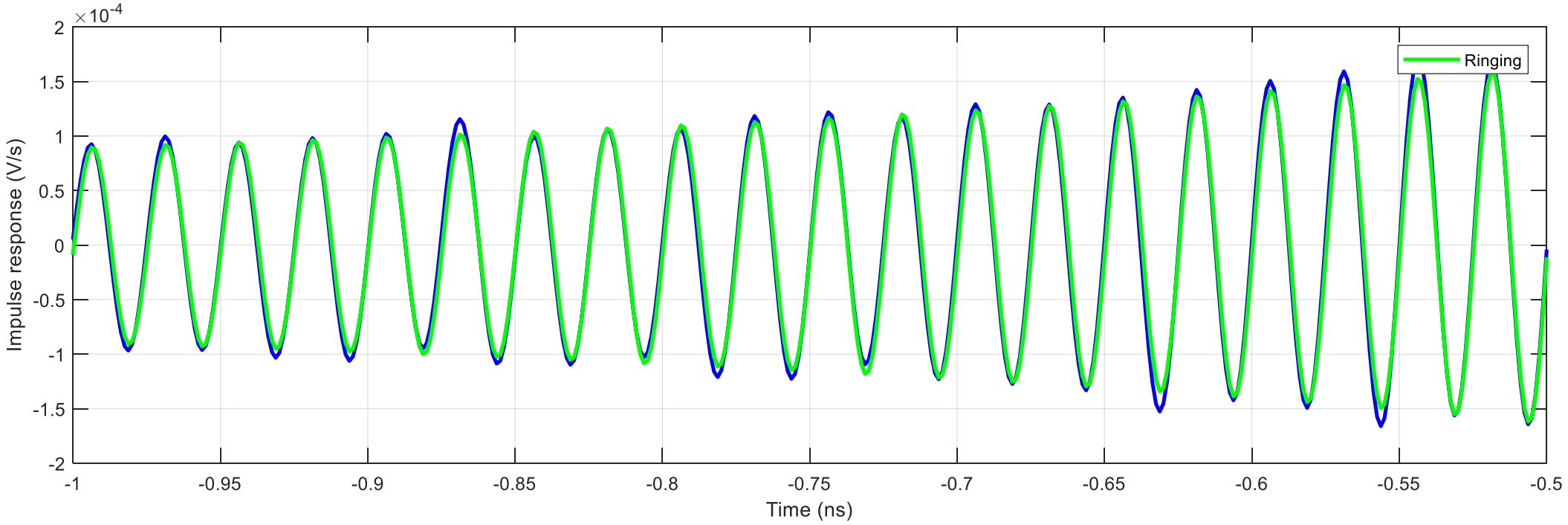


Causality Correction Methods

Correction without removal of numerical non-causalities

$$h_{ringing}(t) = h_{inf}(t) * sinc(t) = \int_{-\infty}^{+\infty} h_{inf}(\tau).sinc(t-\tau).d\tau \approx \sum_{n=1}^{N} H_k.sinc(t-t_k)$$



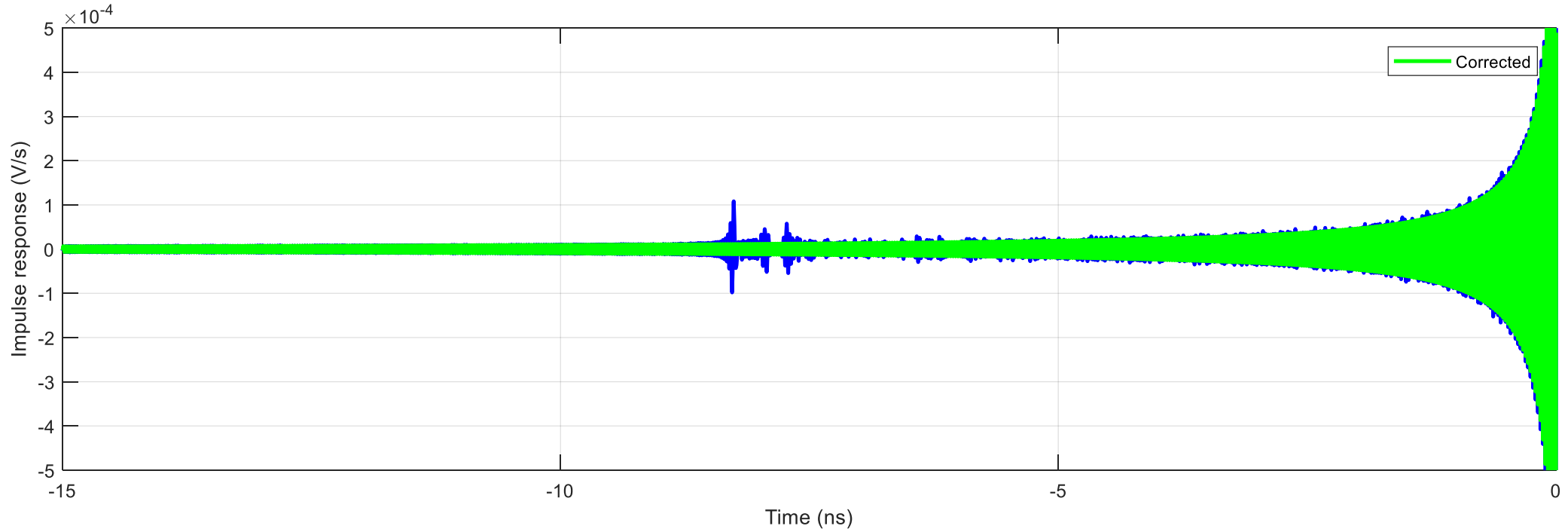
Causality Correction Methods

Correction without removal of numerical non-causalities

$$h_{non\ causal}(t) = h_{original}(t) - h_{ringing}(t)$$

⇨ Enforce causality

⇨ Add ringing



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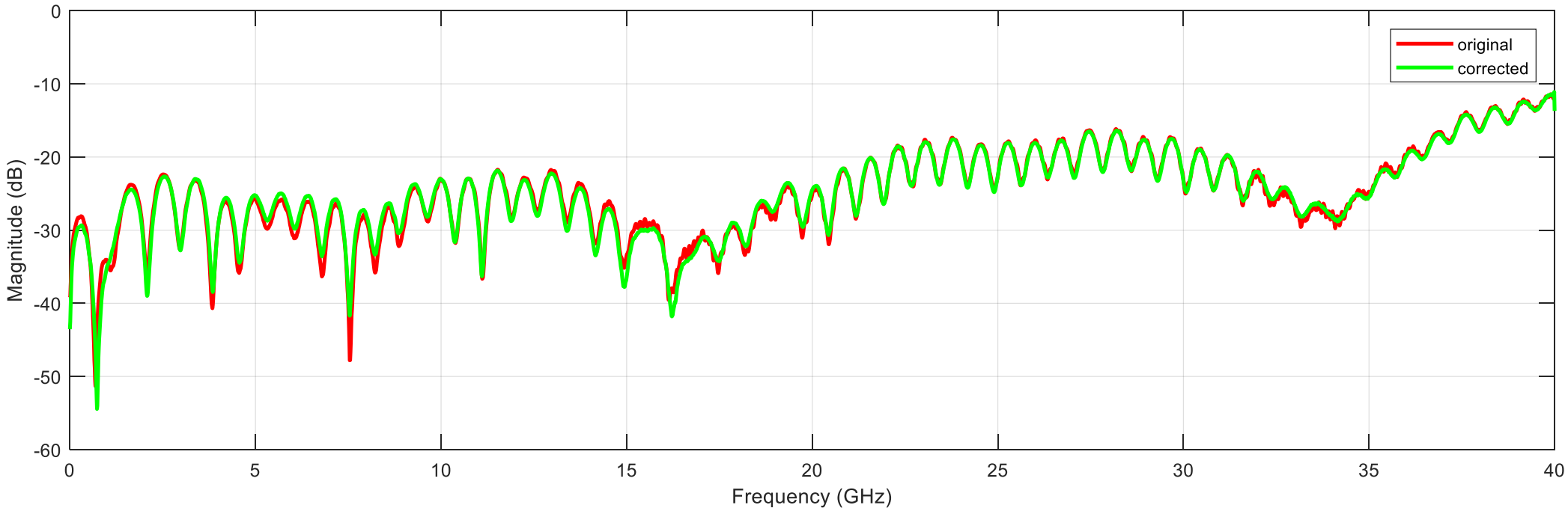

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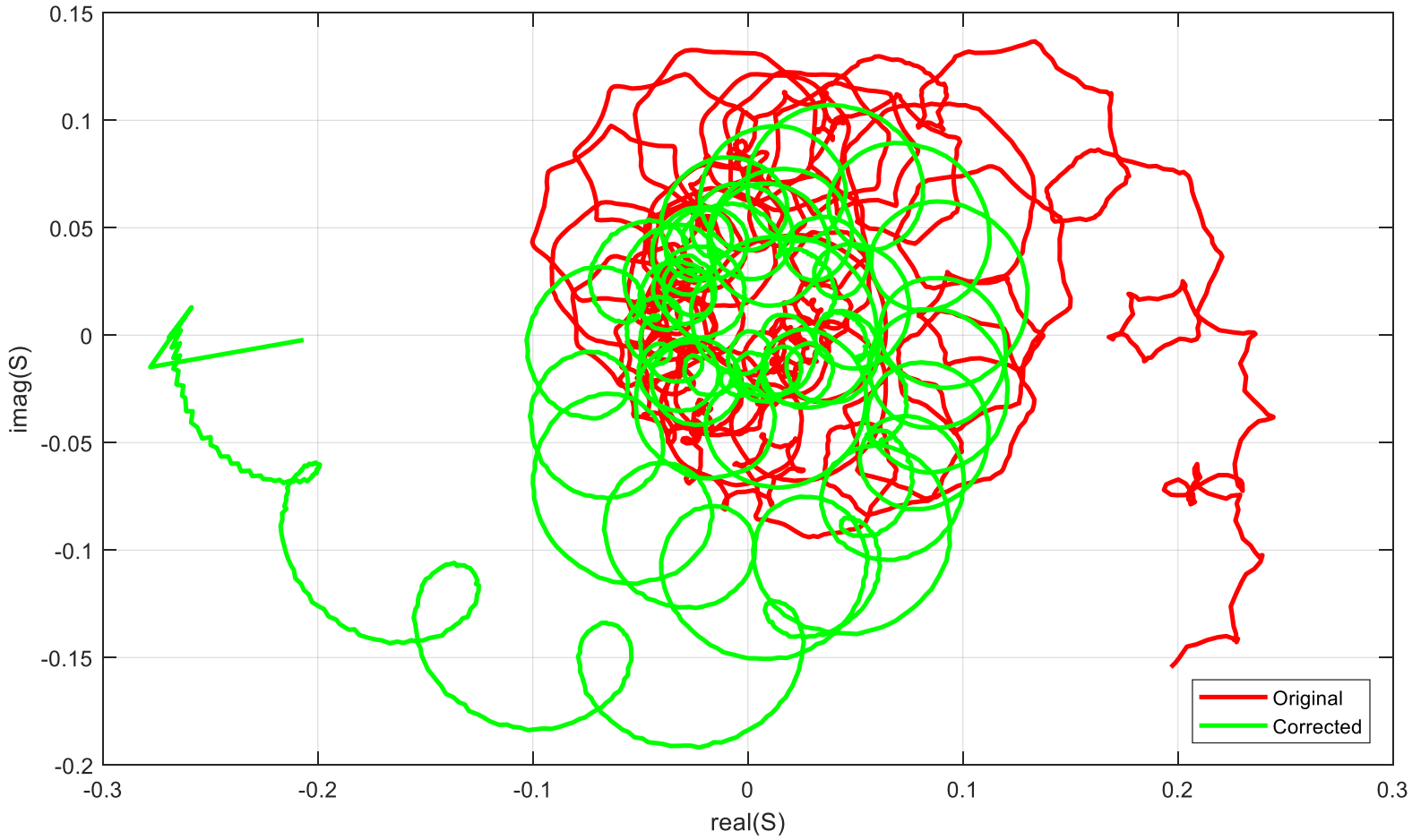
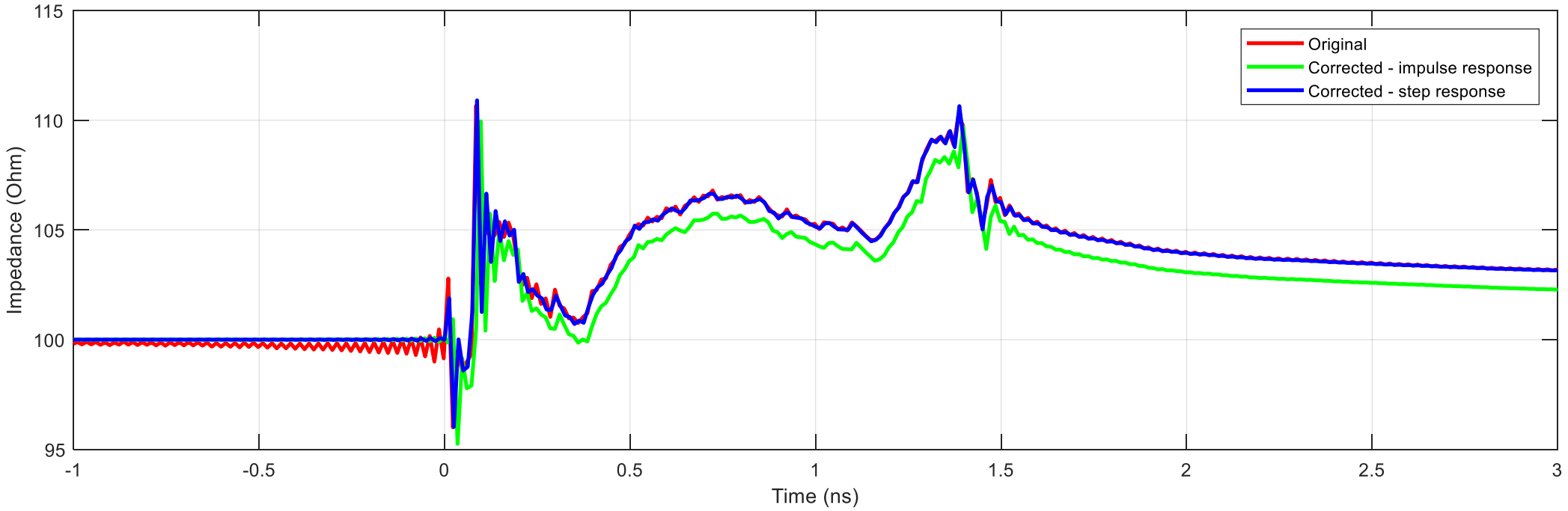


Causality Correction Methods

Correction without removal of numerical non-causalities

- Causality correction caused a DC + impedance shift
- Can be avoided if correction is applied to the step response iso impulse response



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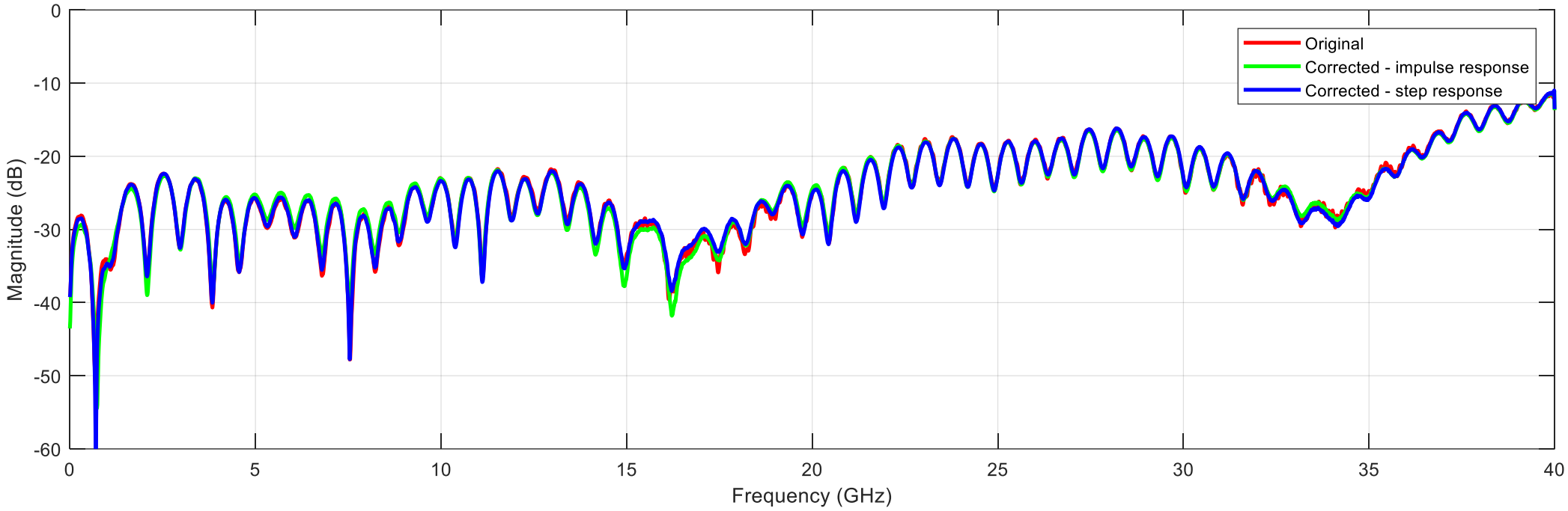
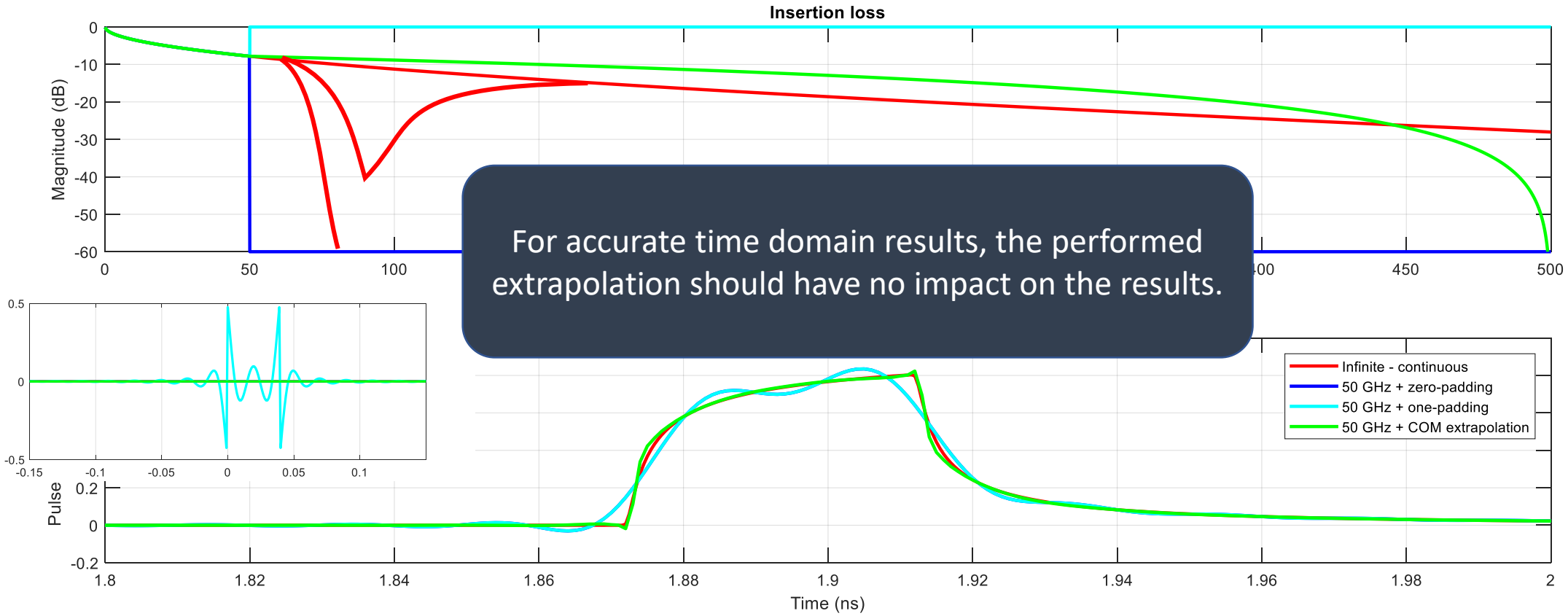


Accurate, Causal Time Domain Results
Removal Numerical Noise without Loss of Accuracy



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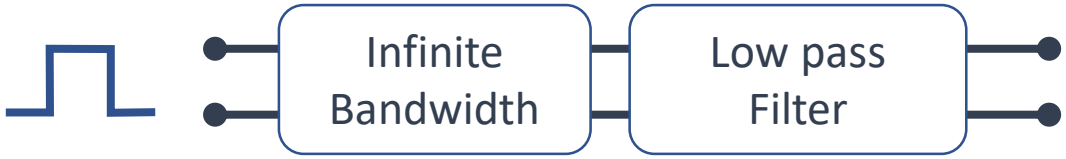
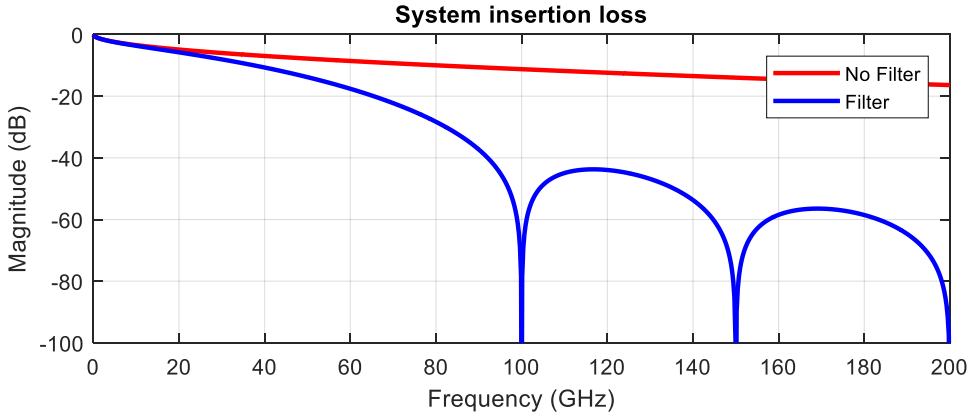
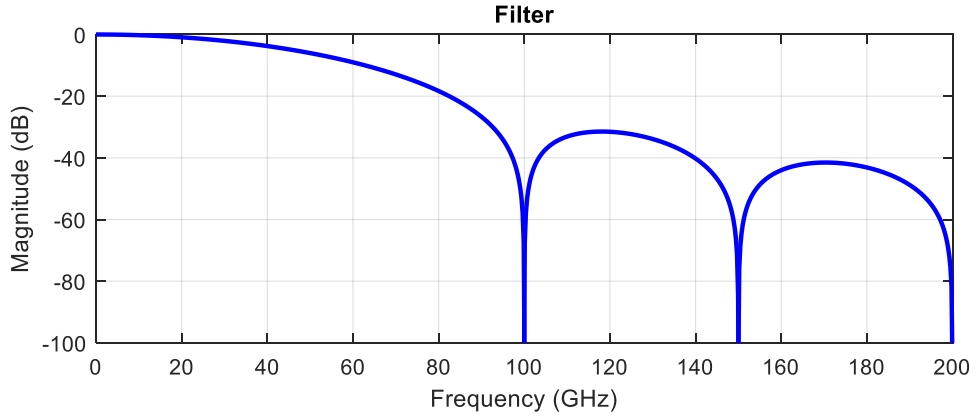
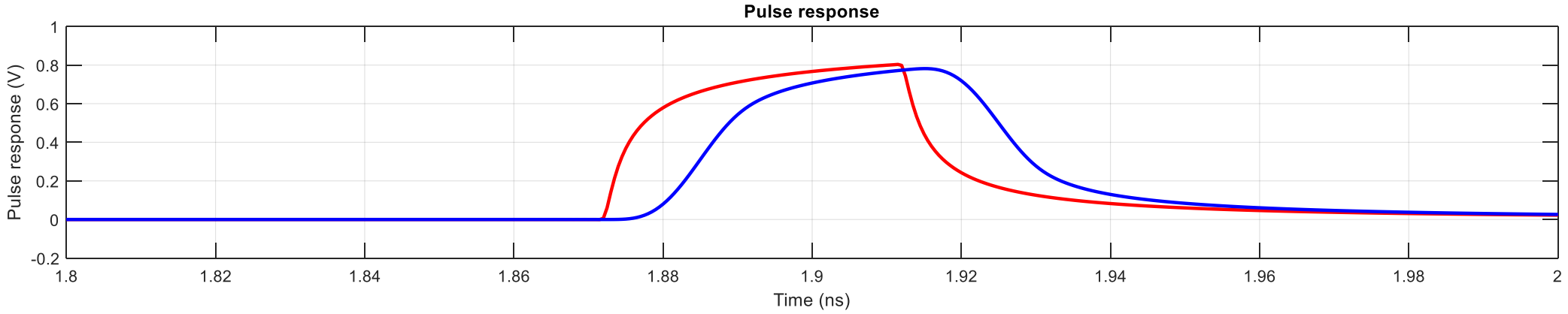
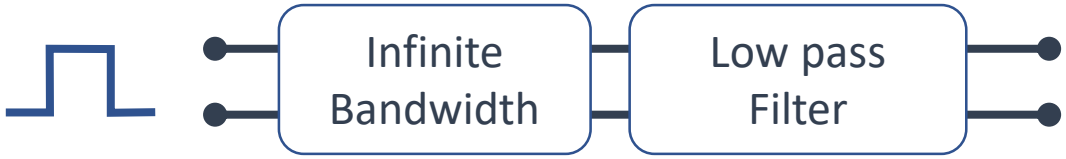


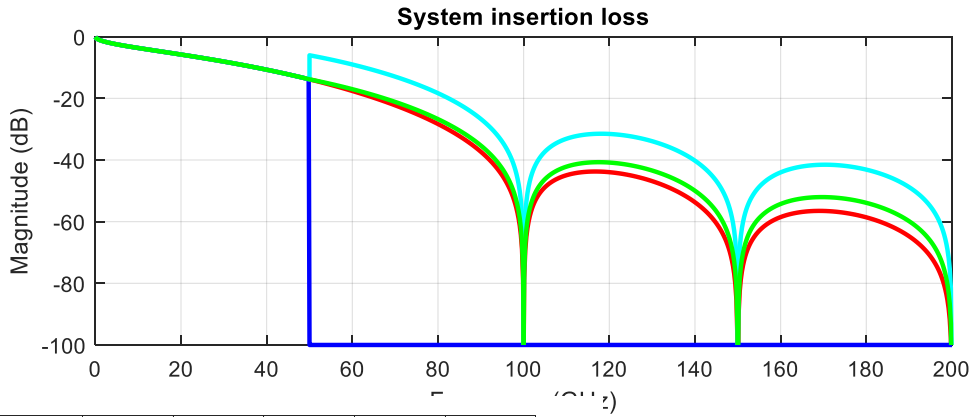


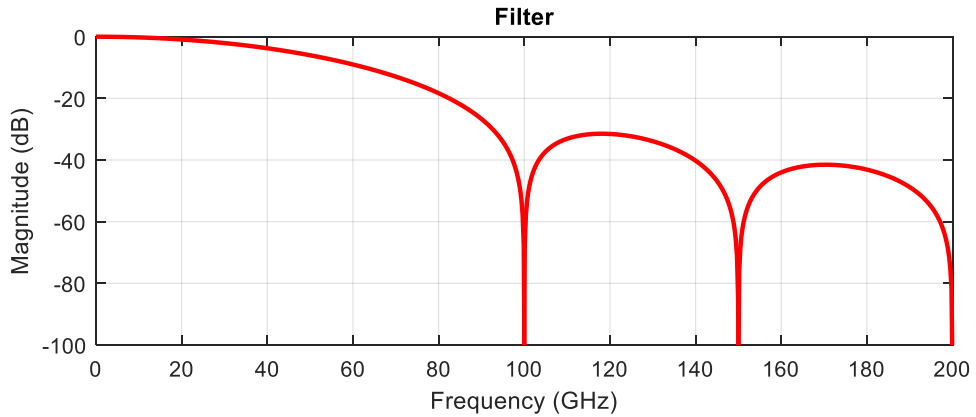


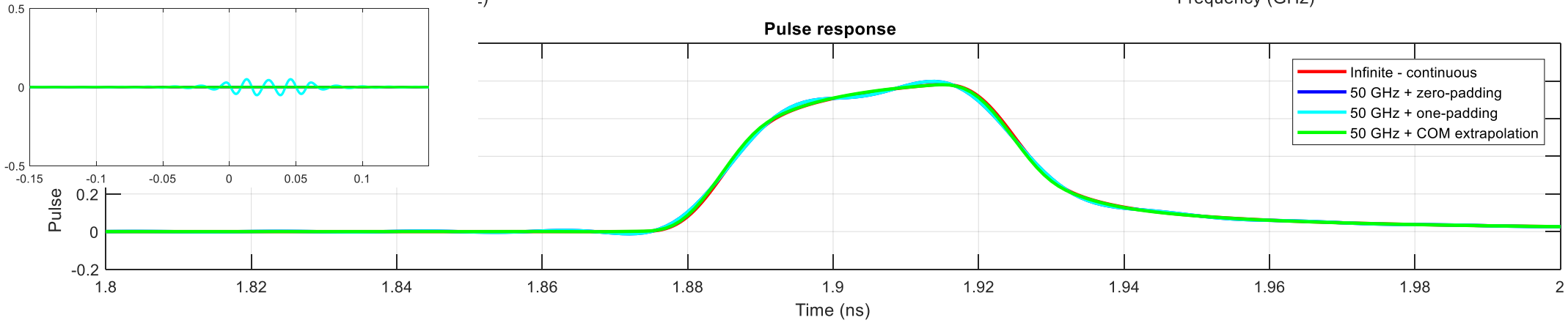


Accurate, Causal Time Domain Results
Removal Numerical Noise without Loss of Accuracy

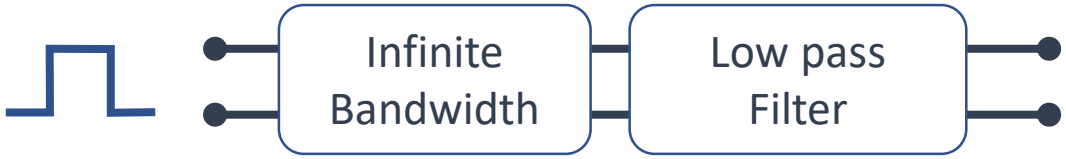


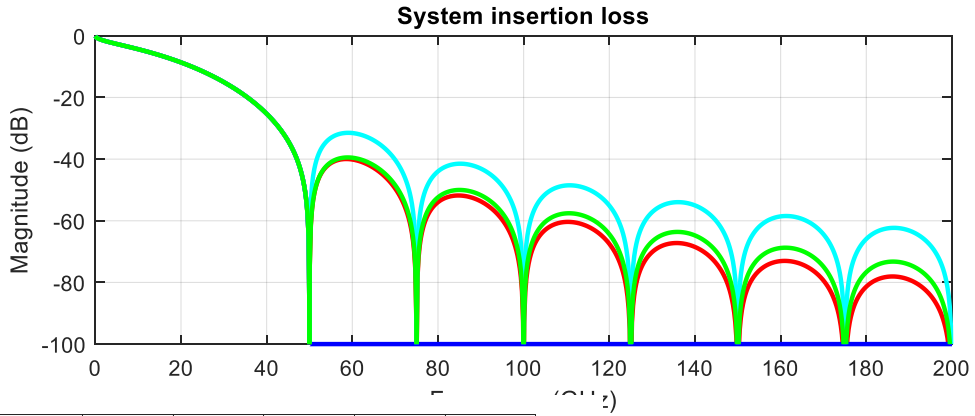


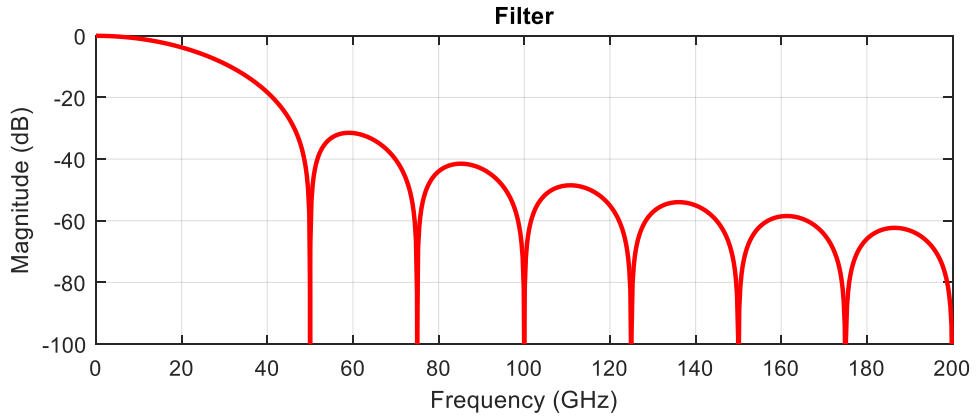


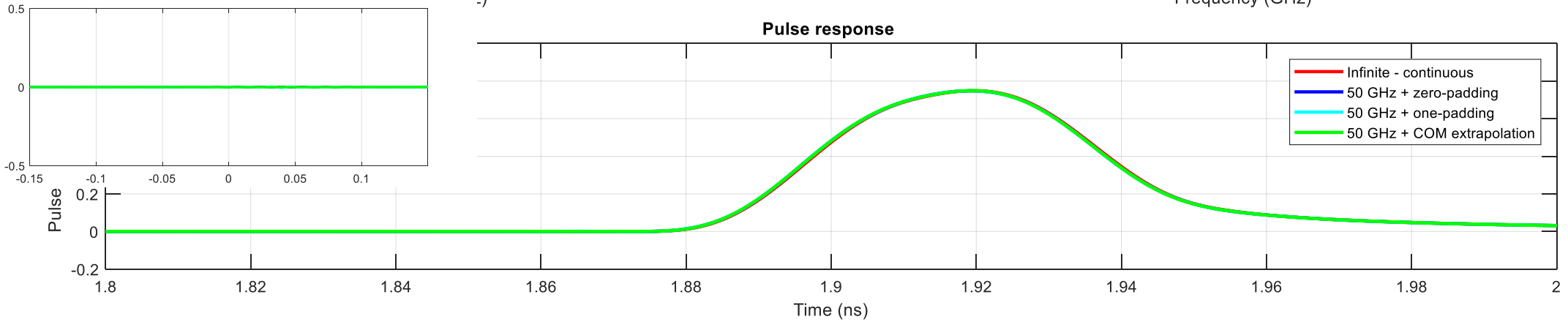


Accurate, Causal Time Domain Results
Removal Numerical Noise without Loss of Accuracy

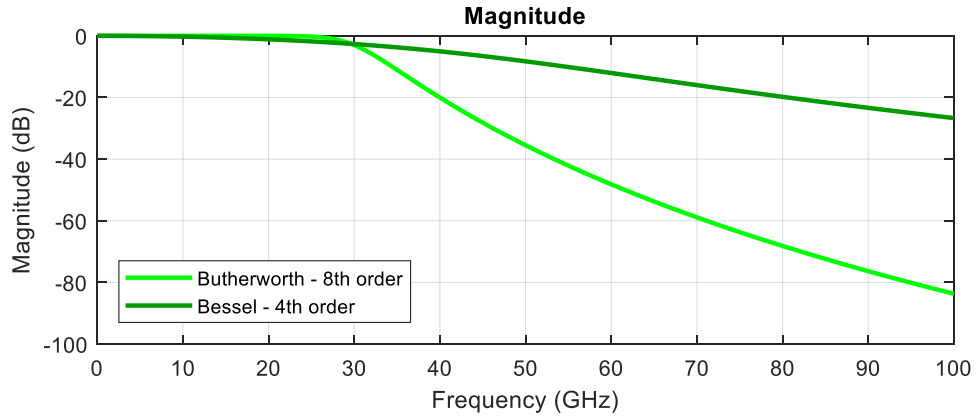


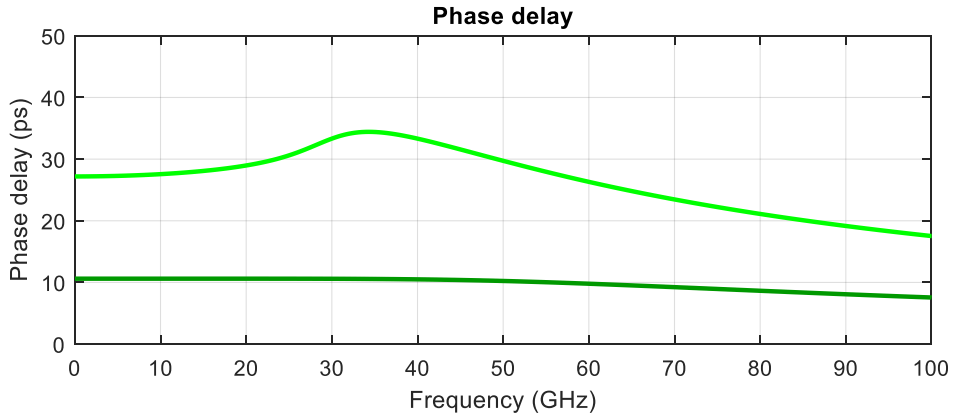


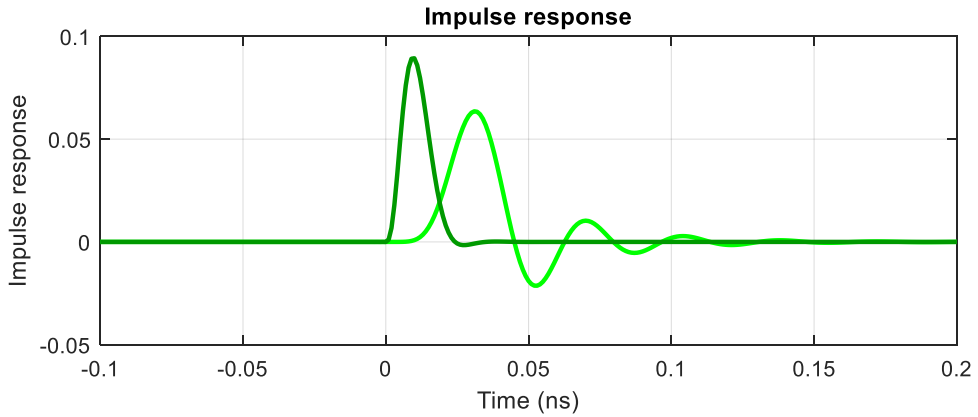


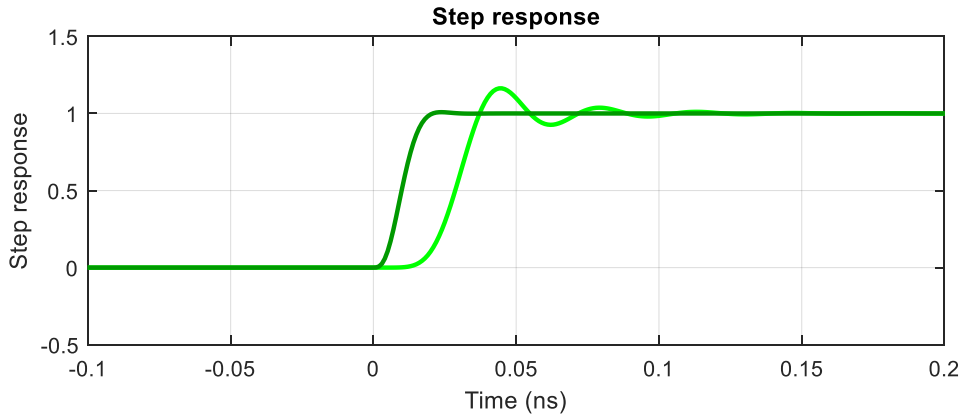


Low Pass Filters

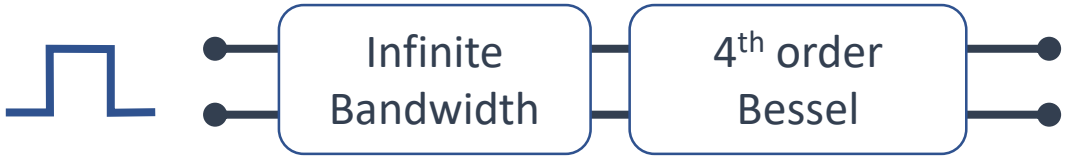


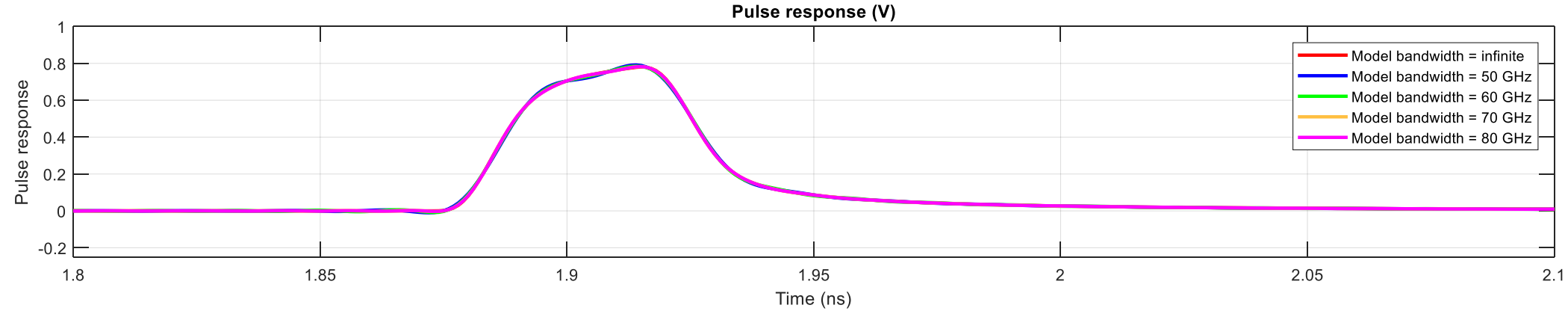


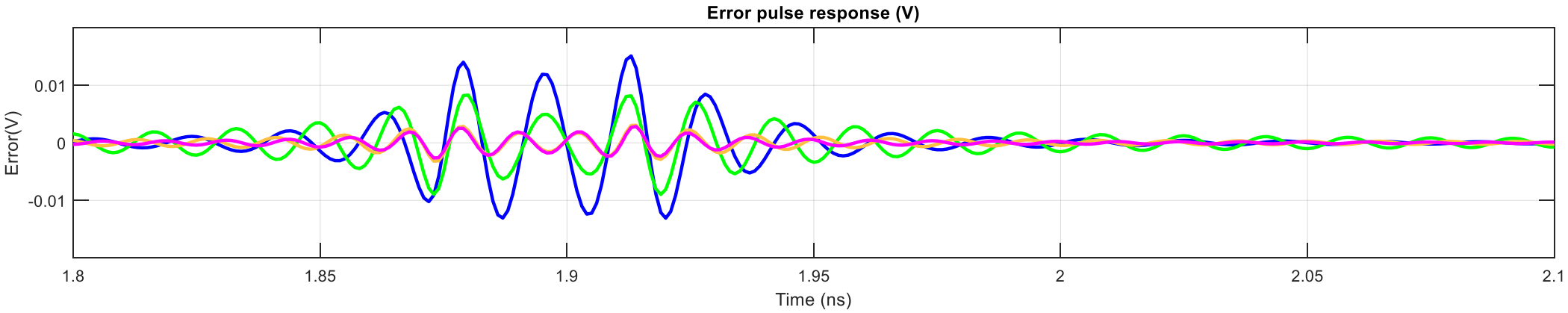




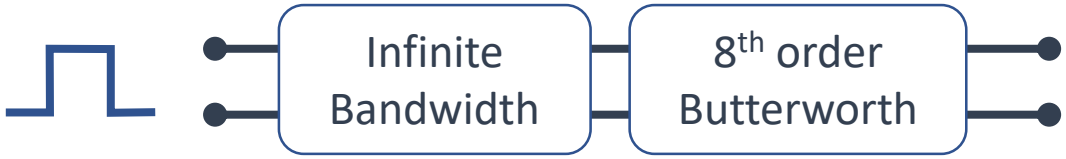
Accurate, Causal Time Domain Results
Removal Numerical Noise without Loss of Accuracy

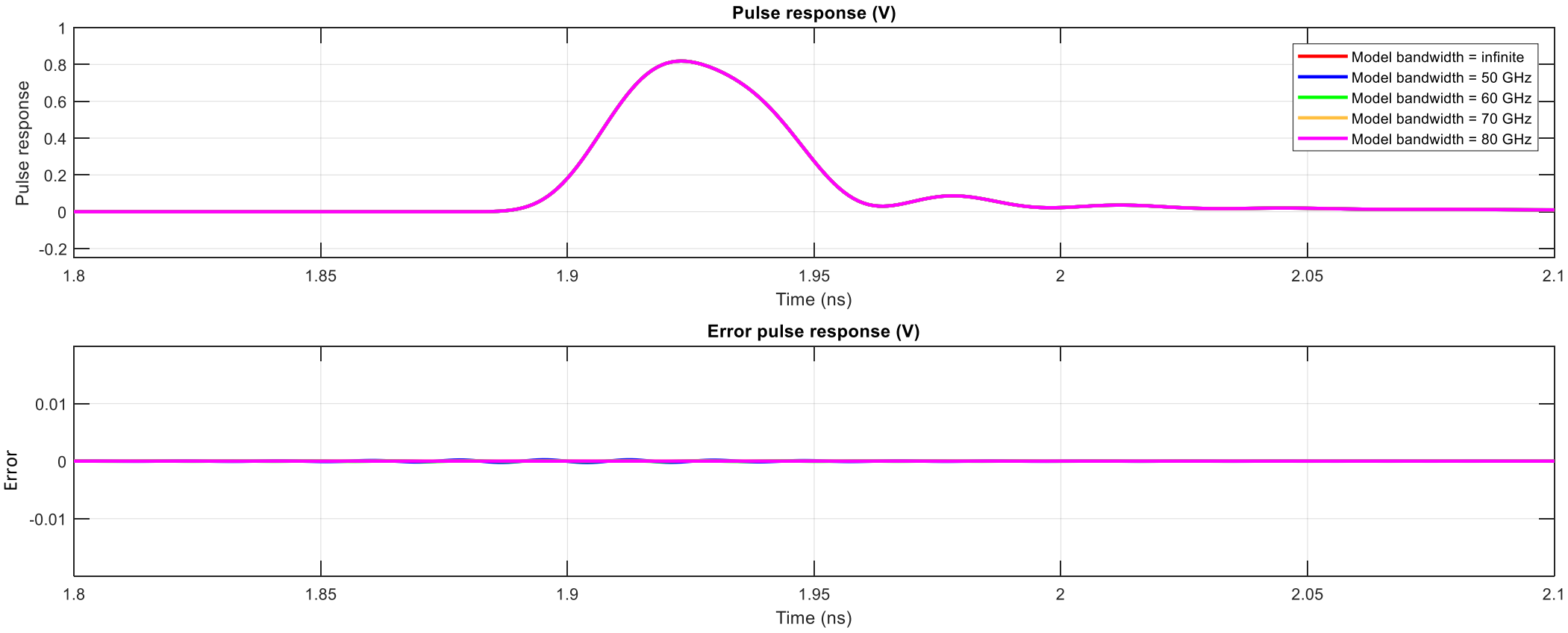






Accurate, Causal Time Domain Results
Removal Numerical Noise without Loss of Accuracy





Summary

- Non-causalities can be grouped in physical and numerical
  - Bandwidth limitation and discretization lead to numerical non-causalities
  - Inaccurate modeling and measurements lead to physical non-causalities
- Causality enforcement methods should only remove physical non-causalities
  - Numerical non-causalities are not related to the quality of a model
  - Causality correction methods should have 3 steps
    - Remove numerical non-causalities
    - Remove physical non-causalities
    - Add numerical non-causalities
  - Hilbert causality correction remove numerical and physical non-causalities, and should avoided
- Presence of a bandwidth limiting component in a channel (low pass filter) makes it possible to obtain accurate time domain results with bandwidth limited S-parameters
  - System bandwidth (including filters) + required accuracy defined the minimum bandwidth a model should have
- Causality correction should be avoided: find root cause and fix it!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IEEE 370 Causality metrics

Causality metrics

- IEEE 370-2020 standard: IEEE Standard for Electrical Characterization of Printed Circuit Board and Related Interconnects at Frequencies up to 50 GHz

Metric 1: Initial Causality Quality metric

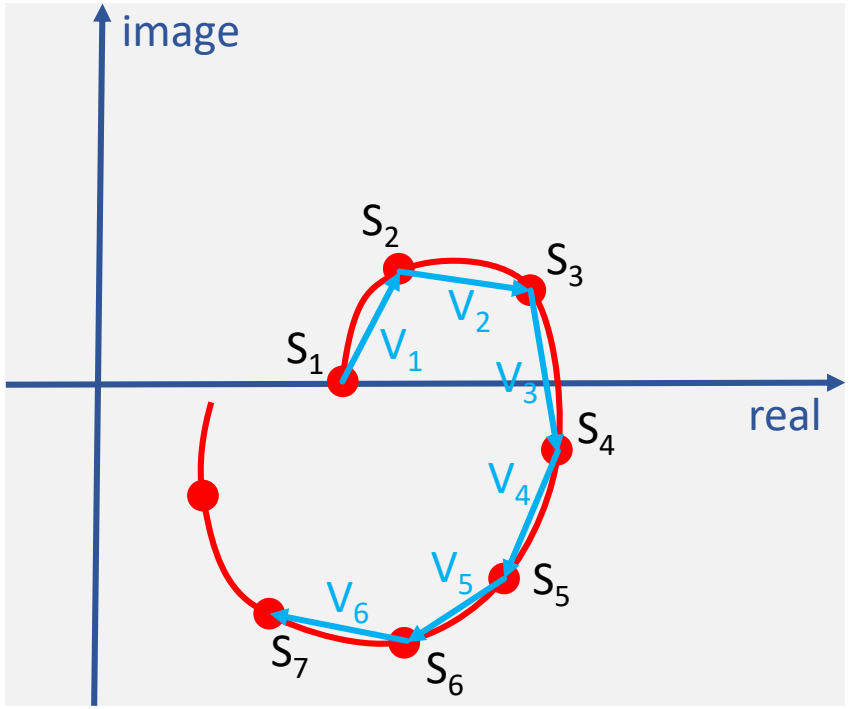
- heuristic, frequency domain check, informal
- $CQM_i = \min(CQM_i^{(j,k)})$ (j,k = 1,… number of ports)
  -]80,100] = Good
  -]50,80] = Acceptable
  -]20,50] = Inconclusive
  - [0,20] = Poor

Metric 2: Application Causality Quality metric

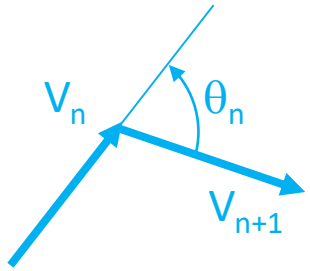
- Application based, time domain check, formal
- $CQM_a = \max(CQM_a^{(j,k)})$ (j,k = 1,… number of ports)
  - [0 mV, 5 mV[= Good
  - [5 mV, 10 mV[= Acceptable
  - [10 mV, 15 mV[= Inconclusive
  - [15 mV, +∞ mV[= Poor

Causality metrics

- Metric 1: Initial Causality Quality Metric: measure for the amount of clockwise rotating frequency points



- $V_n = S_{n+1} - S_n$ (n=1, …, N-1)
- $R_n = V_n \times V_{n+1} = |V_n| . |V_{n+1}| . \sin(\theta_n)$ (n=1, …, N-2)

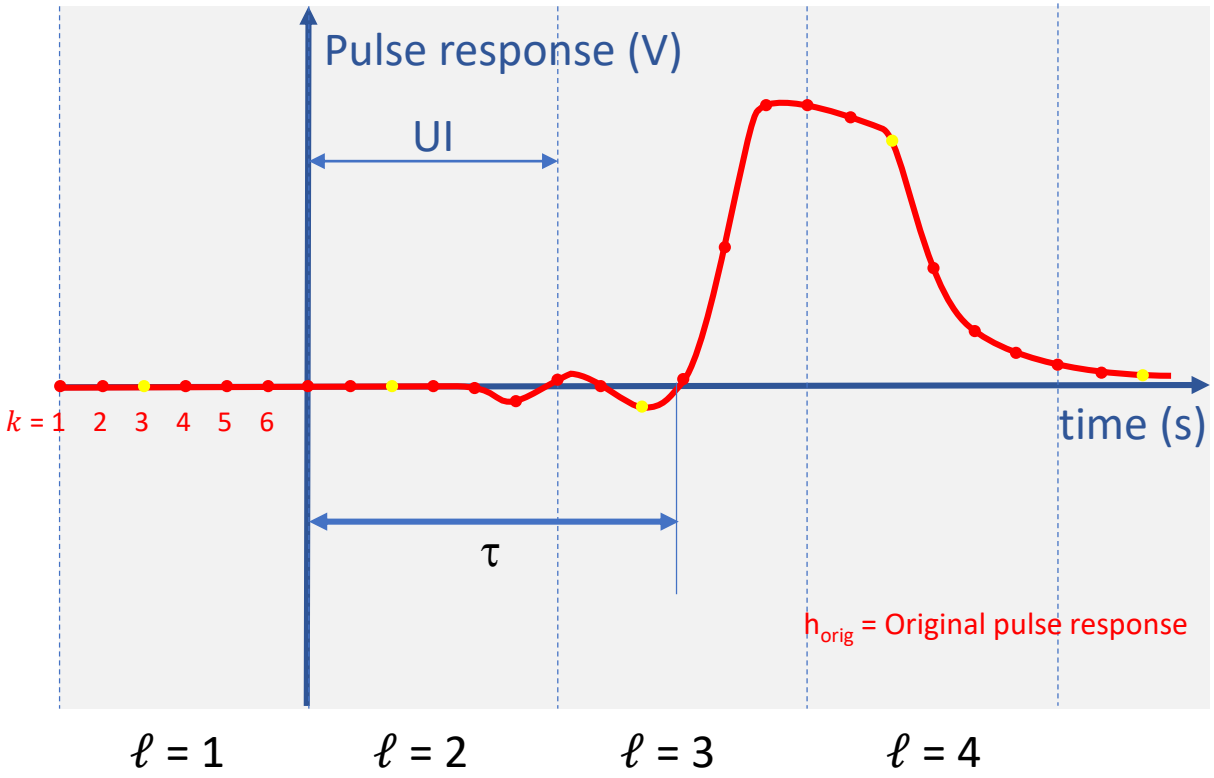


R_n is positive if vectors rotate clockwise, R_n is negative if vectors rotate counterclockwise

- $CQM_i = 100 \times \frac{\sum_{R_n>0} R_n}{\sum_{n=1}^{N-2} |R_n|}$ (n=1, …, N-2)

Causality metrics

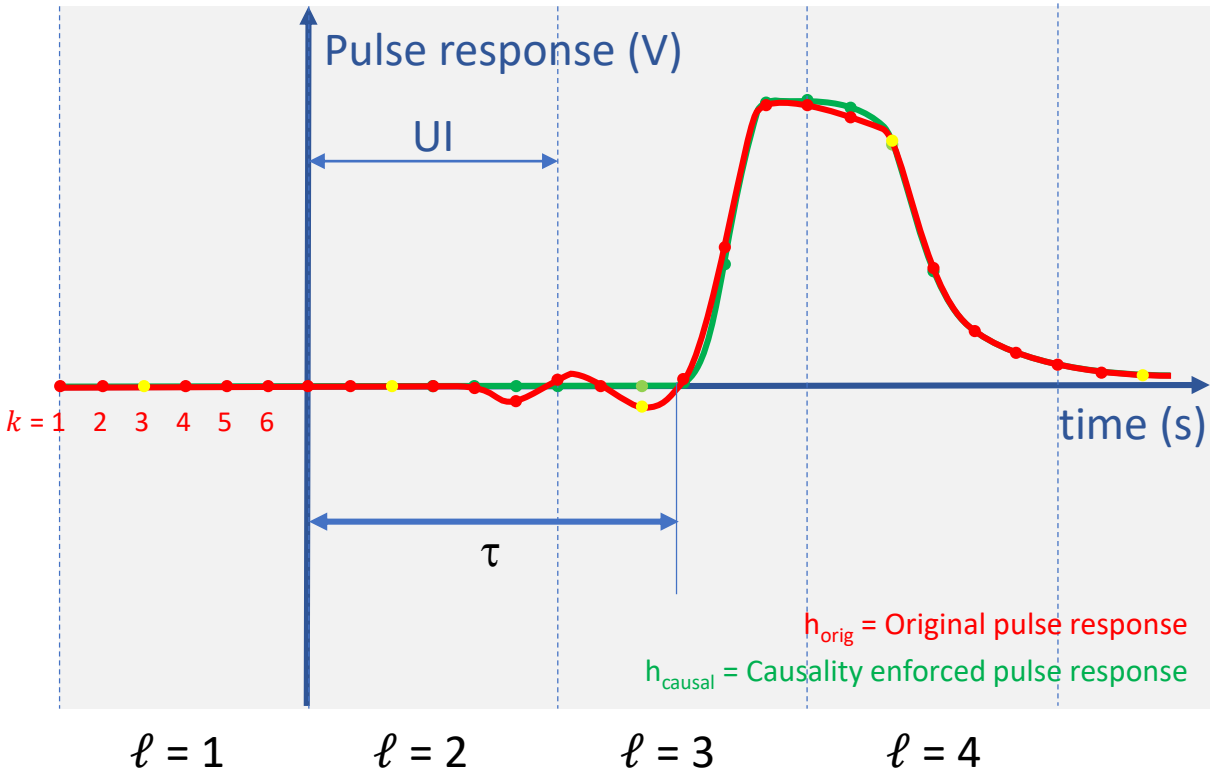
- Metric 2: Application Causality Quality metric



UI = application unit interval
ℓ = unit interval index (ℓ = 1, …, L)
k = sample point index within a UI (k = 1, …, K)
L = number of unit intervals
K = number of sample points within a Unit interval
$t_{\ell, k}$ = time of sample point k in unit interval ℓ
τ = 0 s for reflection and crosstalk S-parameters
$\tau = \tau_{\text{front delay}}$ = estimate for the system response time

Causality Metrics

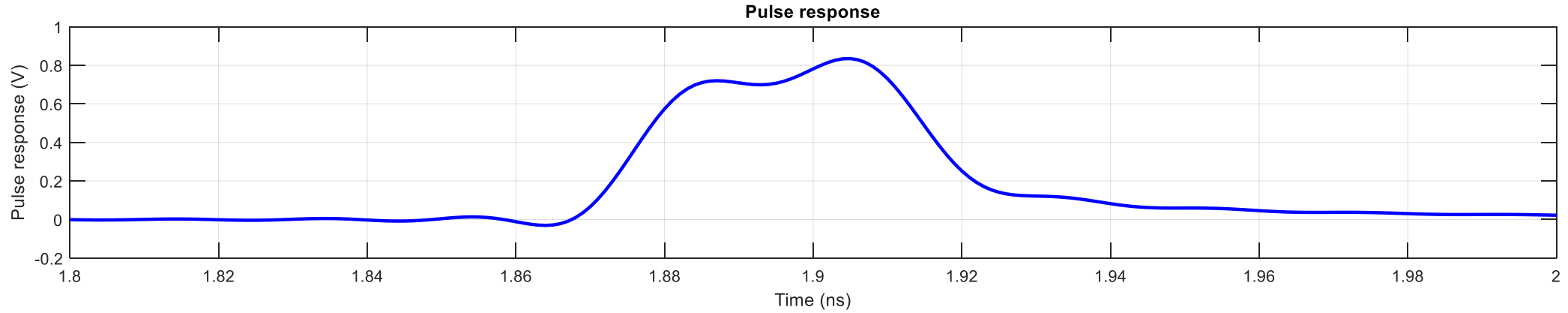
- Metric 2: Application Causality Quality metric

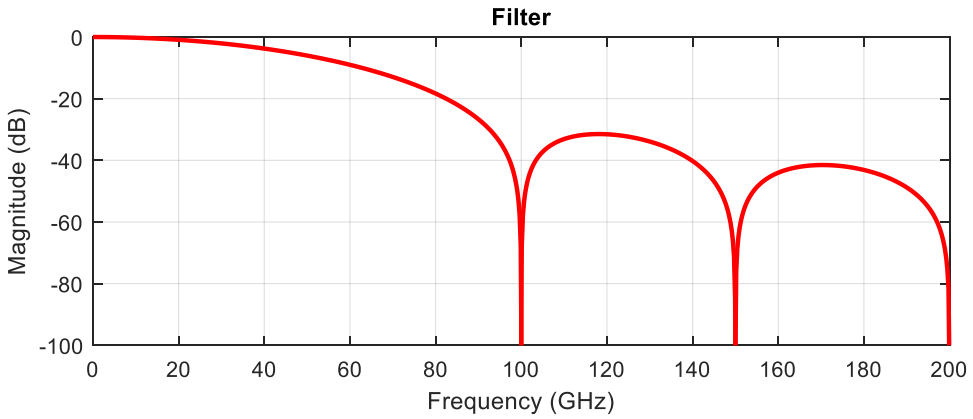


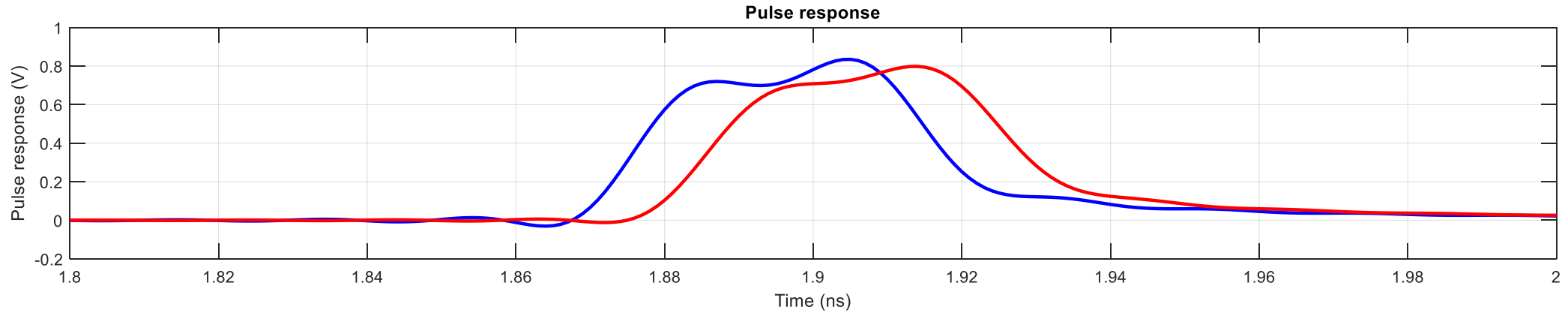
- $\Delta V^{\ell,k} = \left| h_{orig}^{\ell,k} - h_{causal}^{\ell,k} \right|$
- $\Delta V^{k} = \sum_{\substack{\ell=1 \\ t_{\ell,k<\tau}}}^{L} \Delta V^{\ell,k}$
- $CQM_a = \max_{k=1,\ldots,K} \Delta V^{k}$

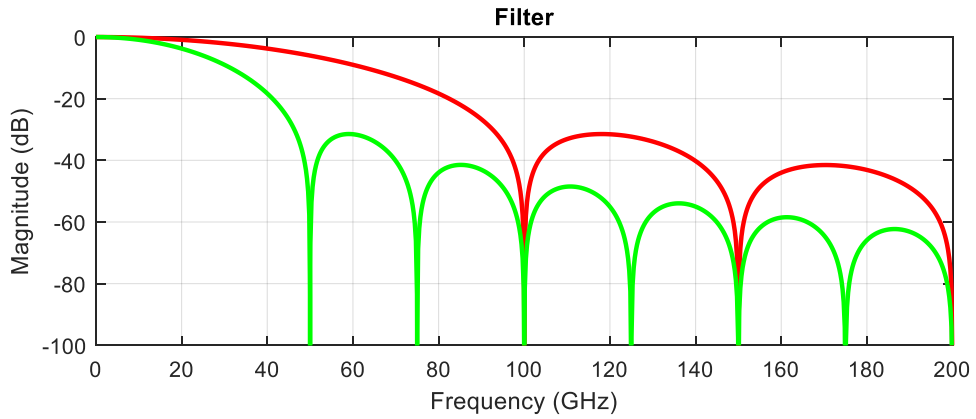
UI = application unit interval
ℓ = unit interval index (ℓ = 1, …, L)
k = sample point index within a UI (k = 1, …, K)
L = number of unit intervals
K = number of sample points within a Unit interval
$t_{\ell,k}$ = time of sample point k in unit interval ℓ
τ = 0 s for reflection and crosstalk S-parameters
$\tau = \tau_{front\ delay}$ = estimate for the system response time

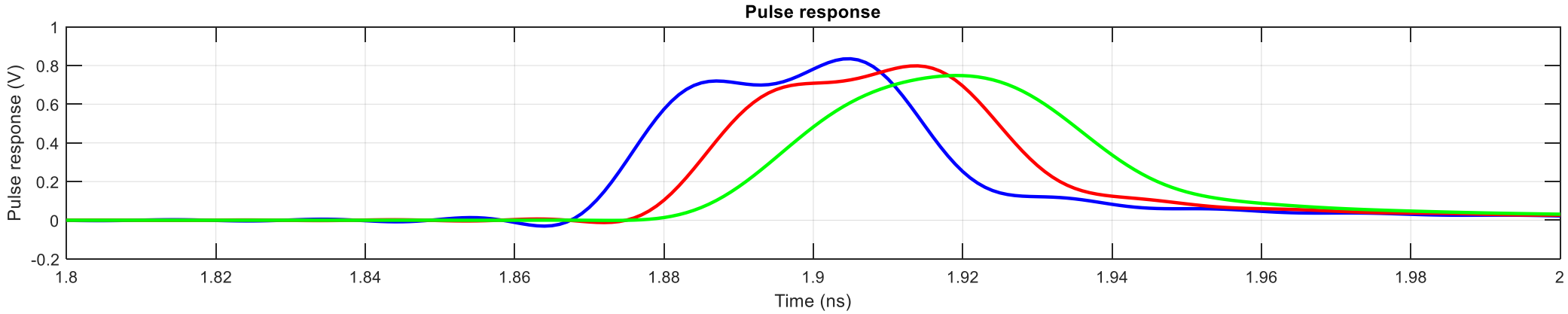
Reduction of ringing by reducing the filter bandwidth





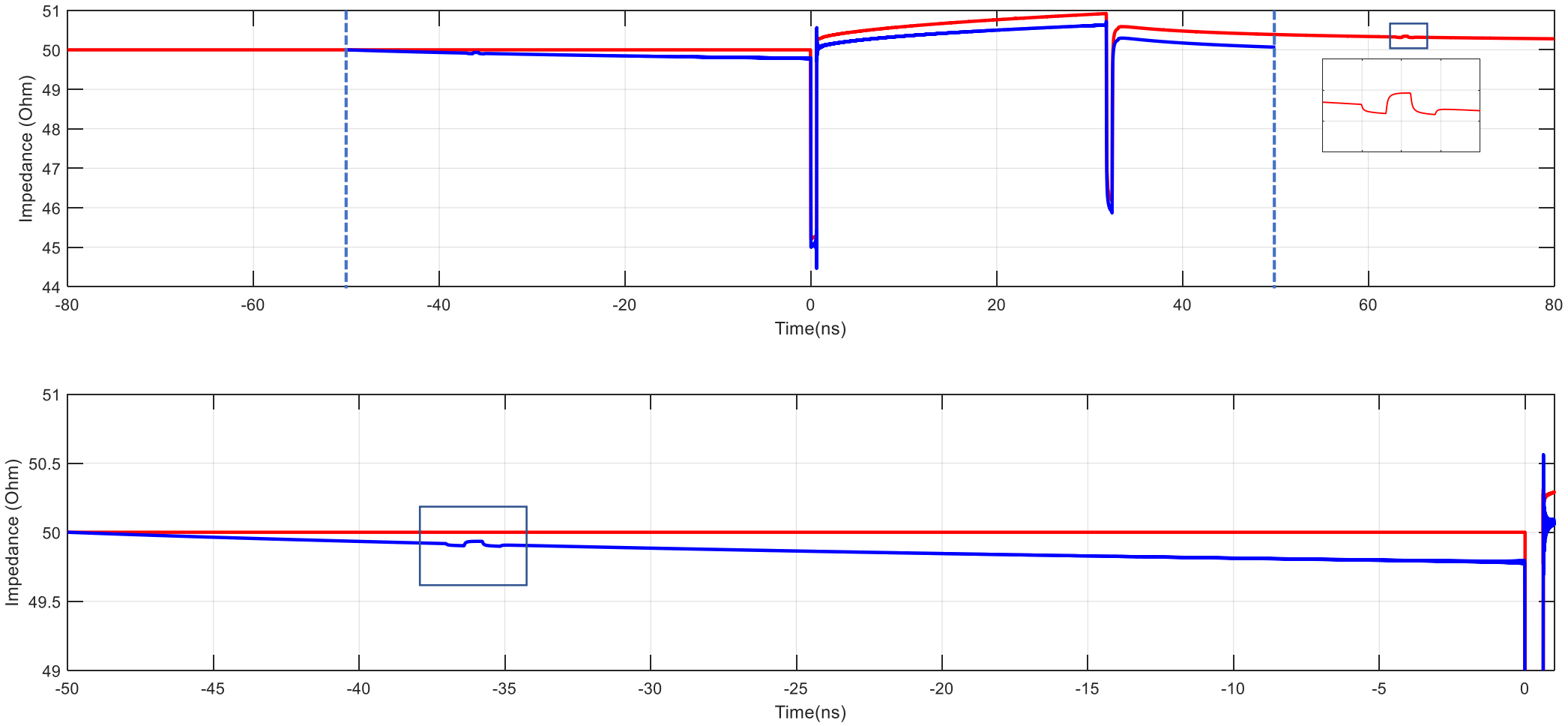




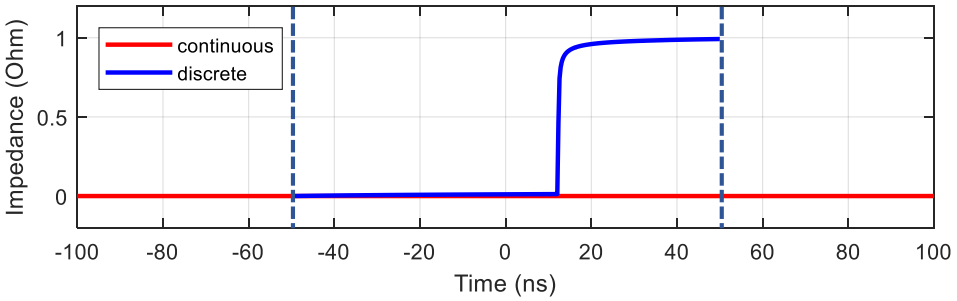


Non causalities caused by discretization/under sampling

Discretization of S-parameters



Discretization of S-parameters Undersampling



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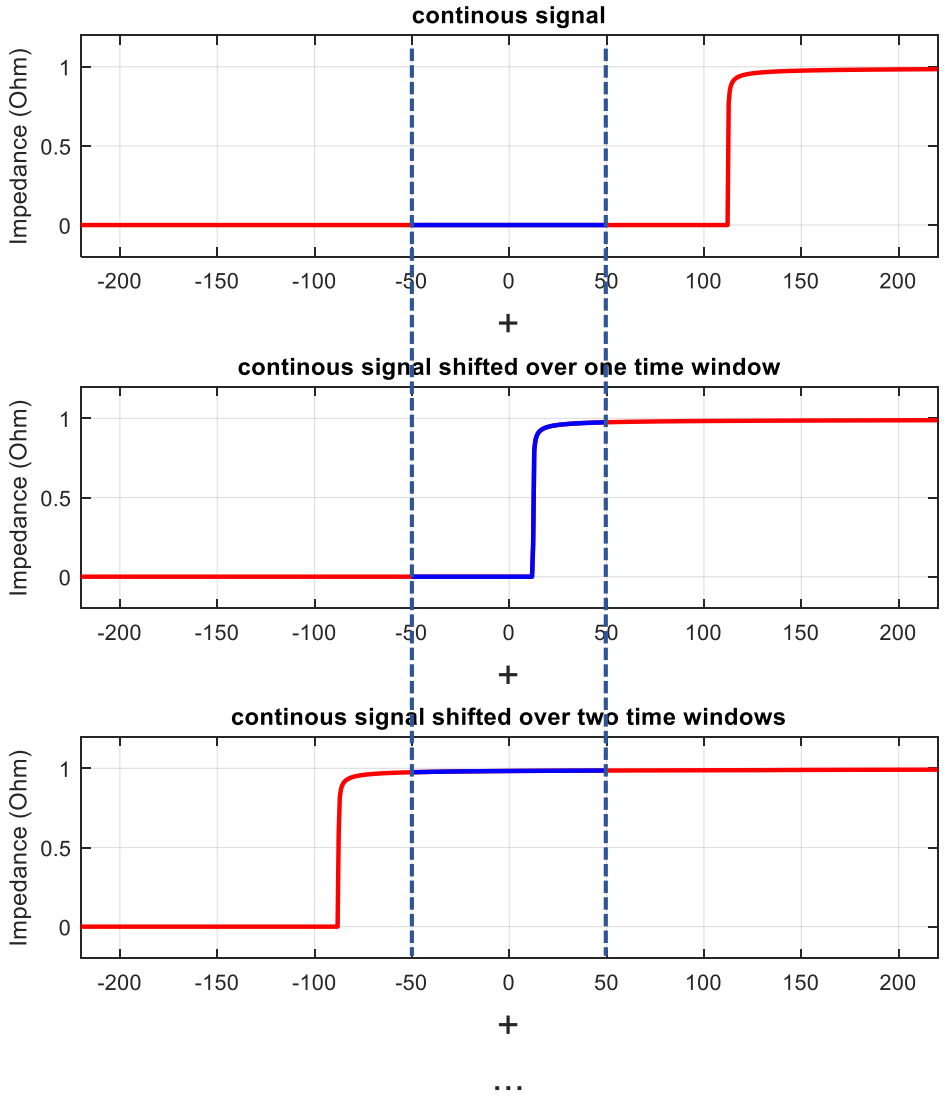
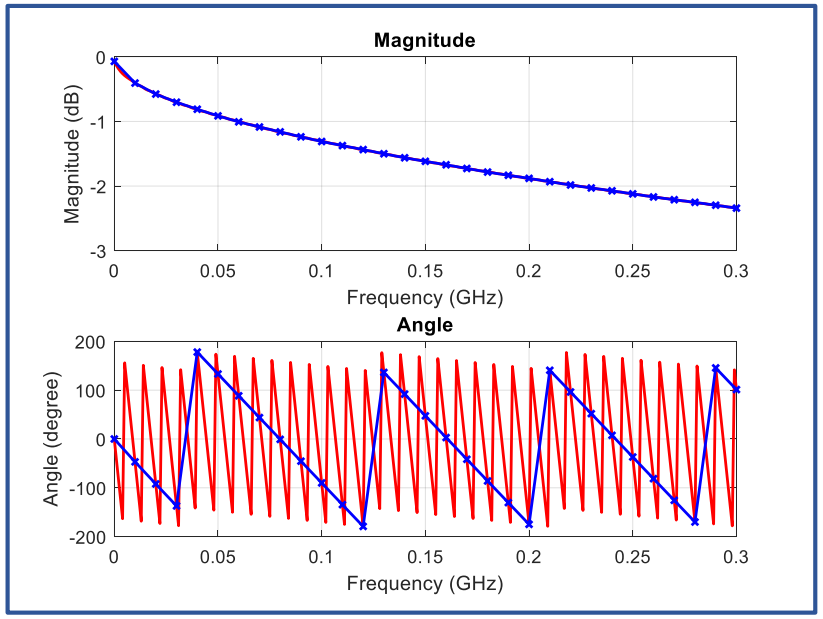
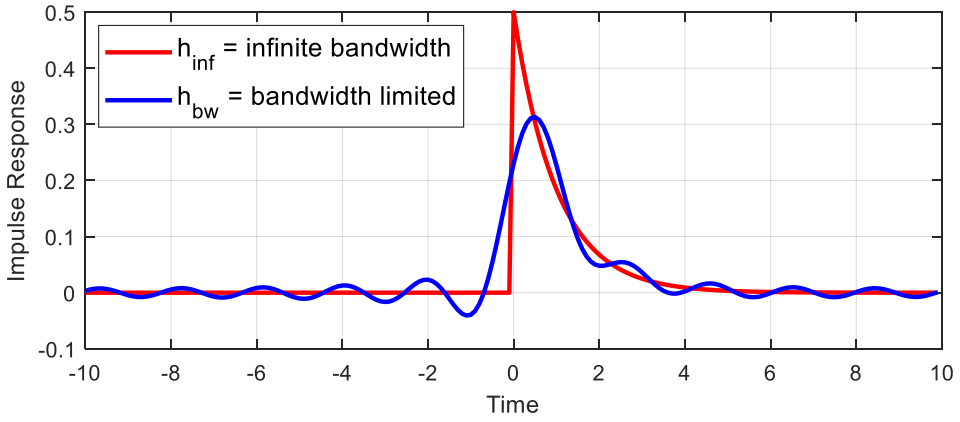




Origin of ringing

Bandwidth limitation of S-parameters



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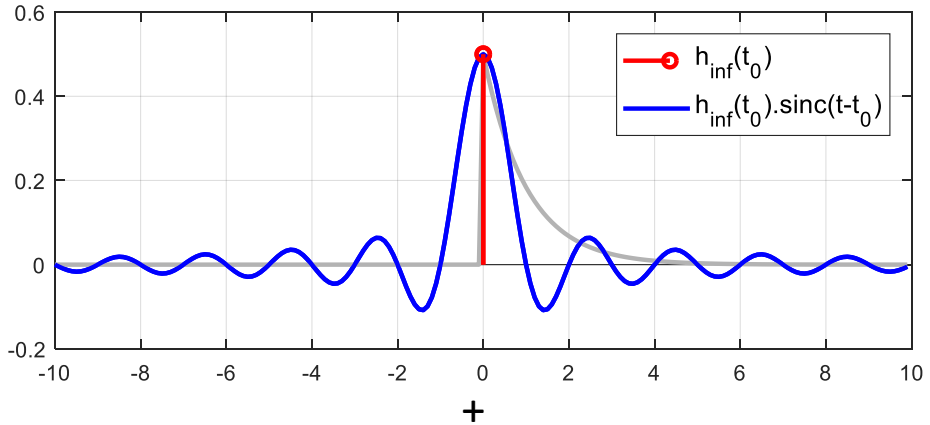

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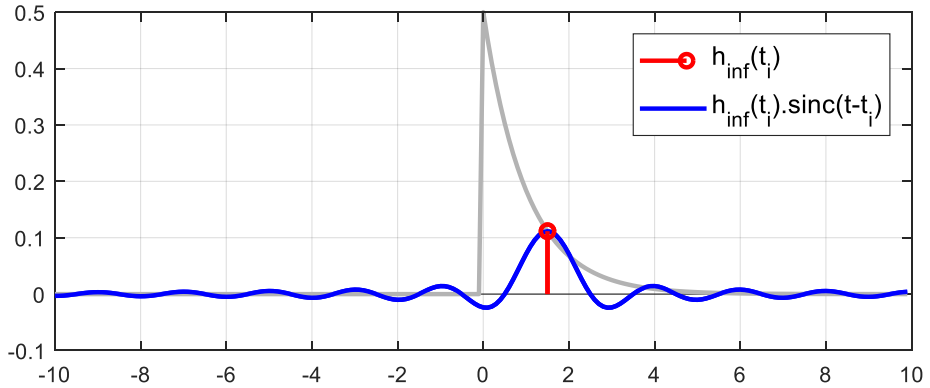


+
...

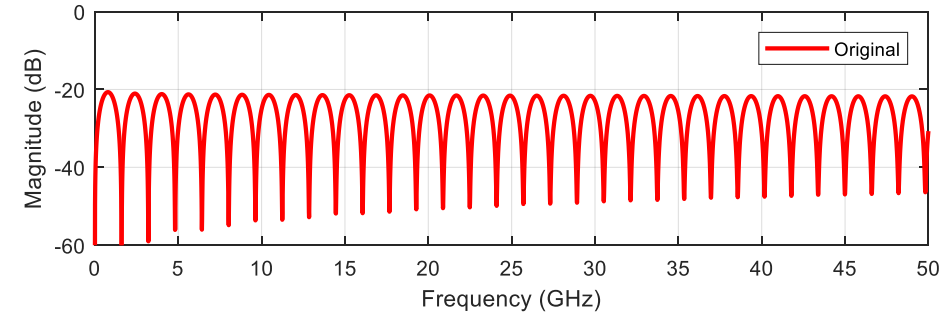
$$h_{bw}(t) = h_{inf}(t) * sinc(t)$$

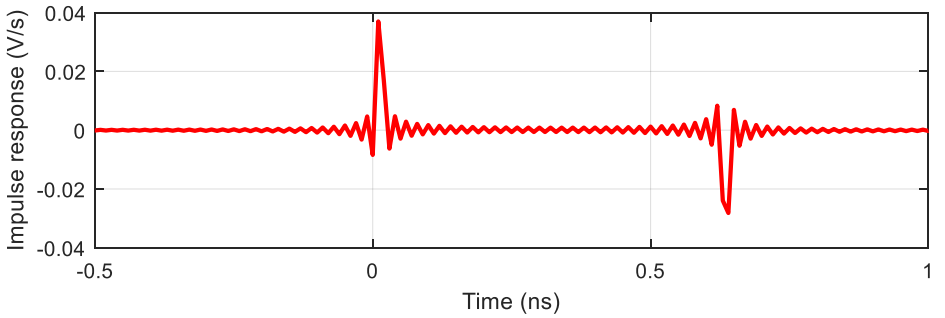
$$= \int_{-\infty}^{+\infty} h_{inf}(\tau).sinc(t-\tau).d\tau$$

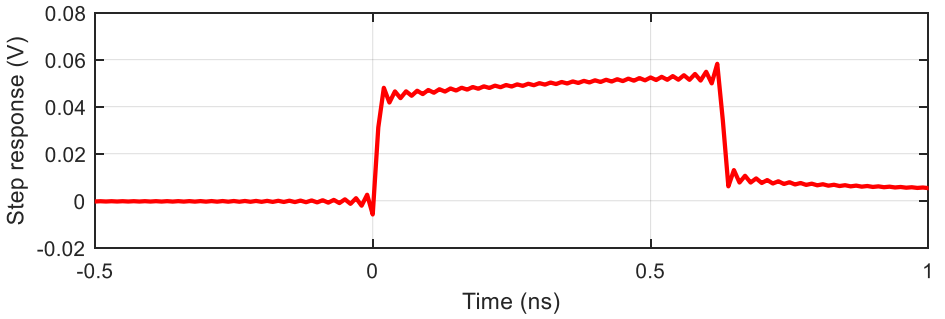
The non-causal behavior is the sum of an infinite number of shifted sinc functions.

Hilbert Amp-Phase

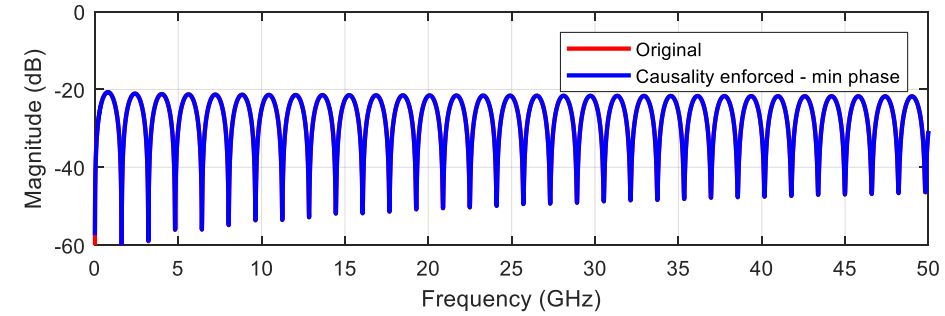
Causality correction methods
Hilbert – amplitude - phase

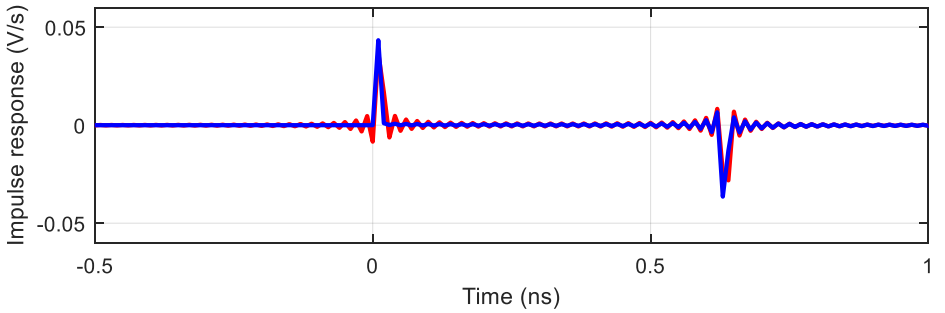


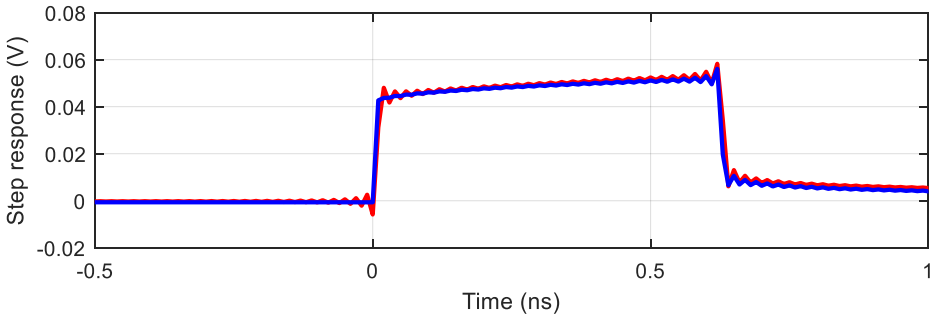




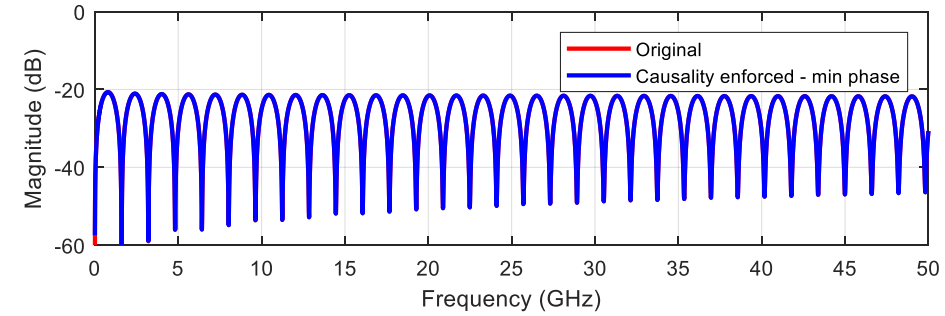
Causality correction methods
Hilbert – amplitude - phase

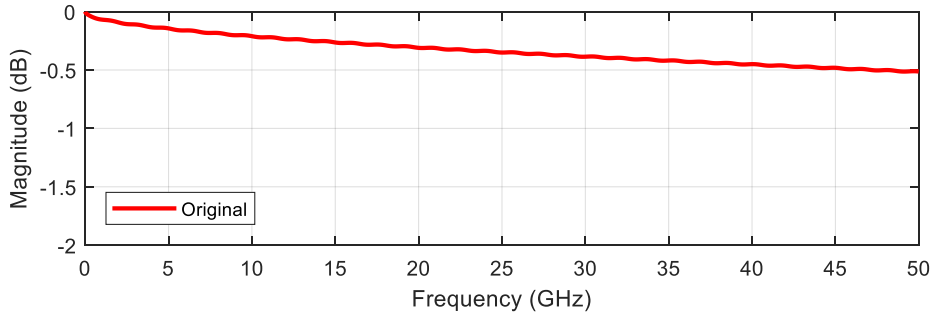


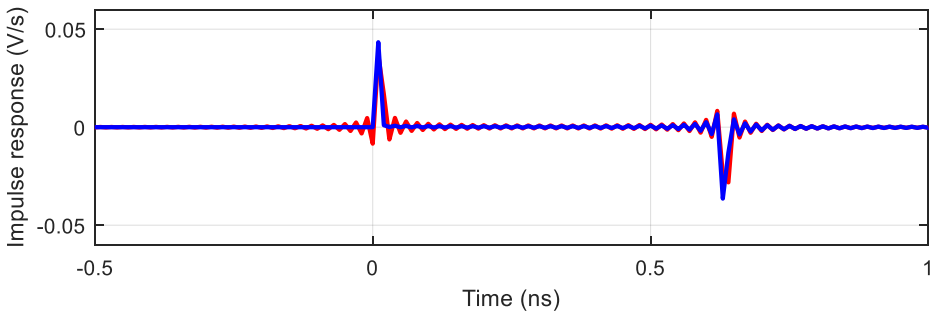


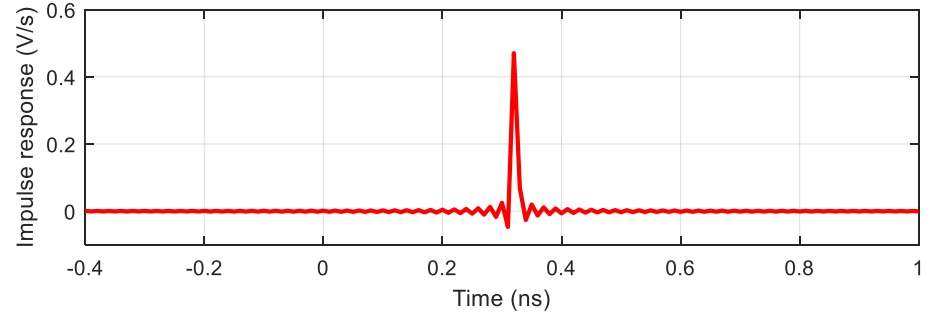


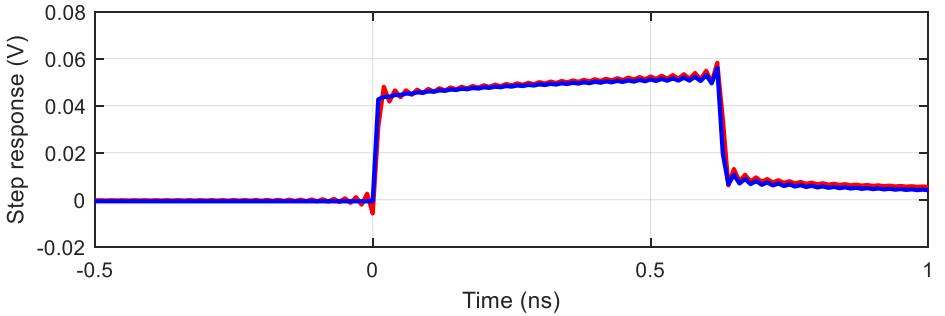
Causality correction methods
Hilbert – amplitude - phase

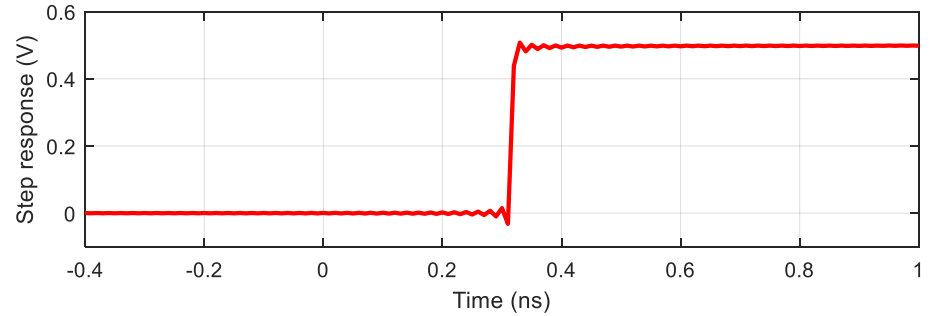




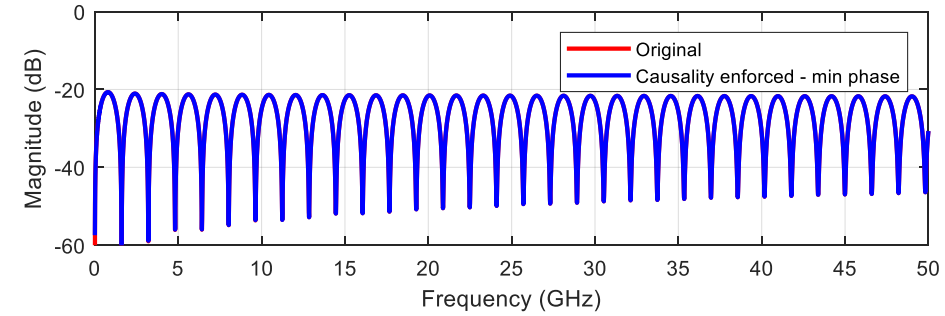


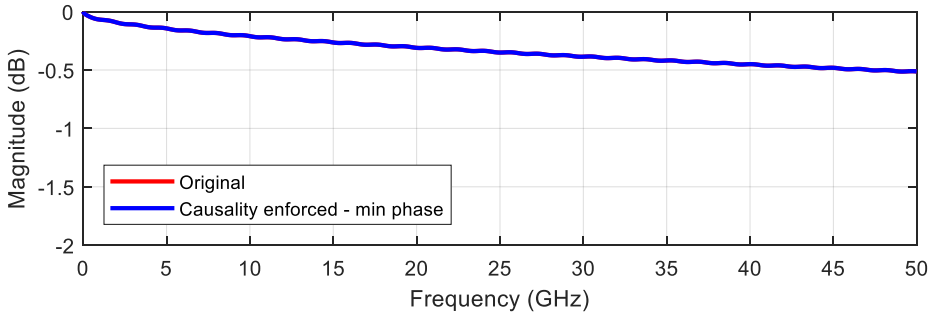


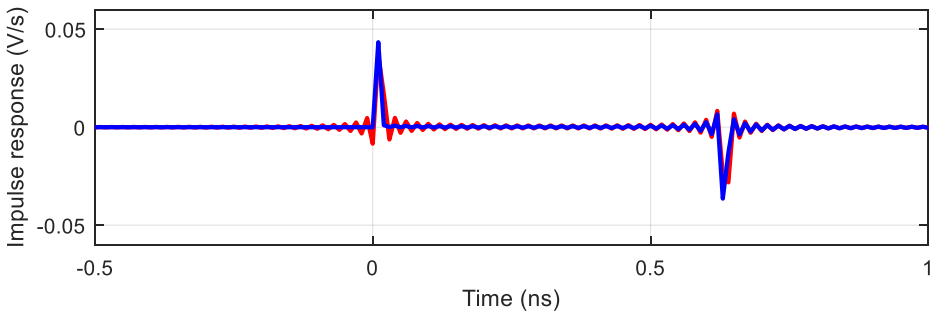


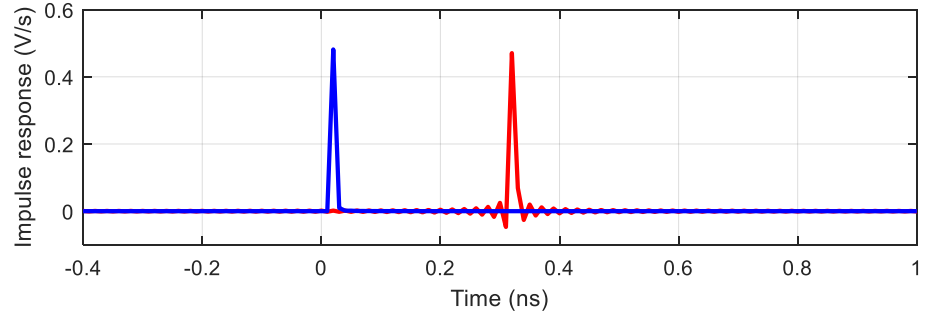


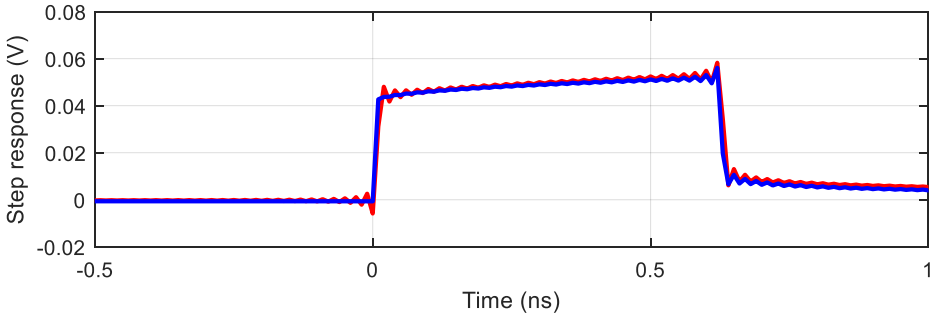
Causality correction methods
Hilbert – amplitude - phase

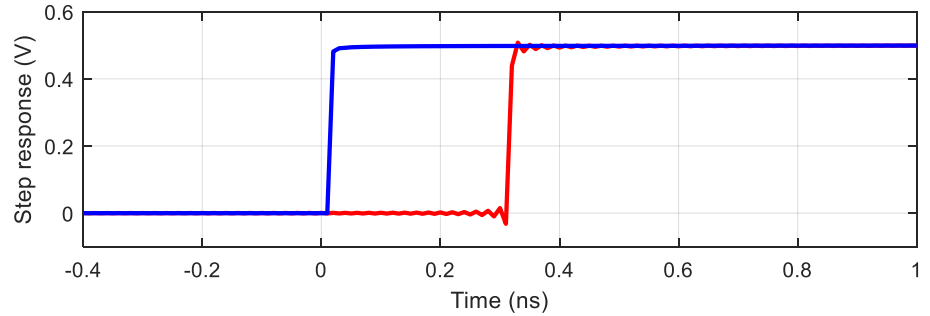




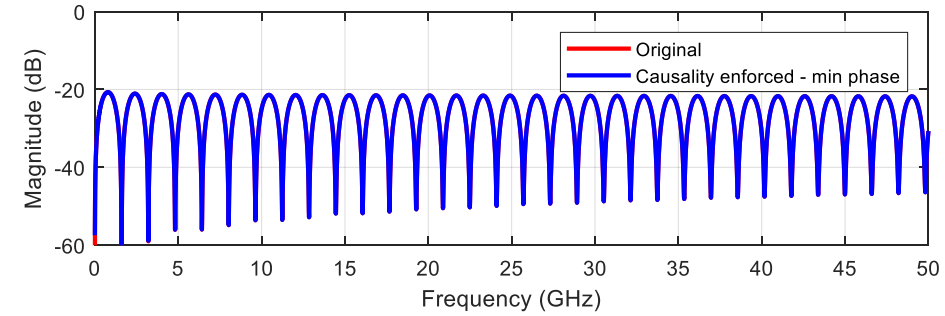


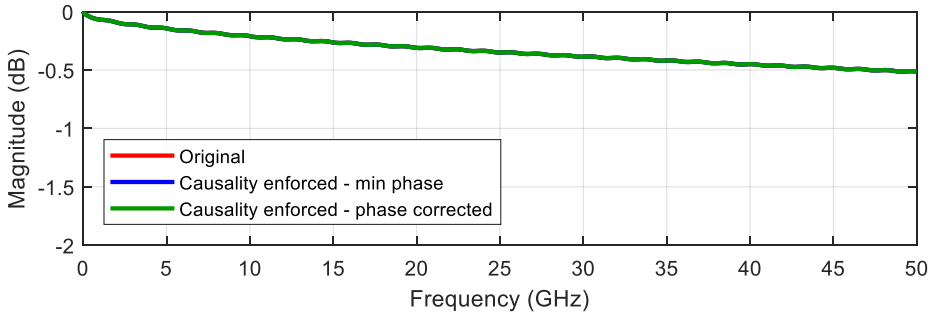


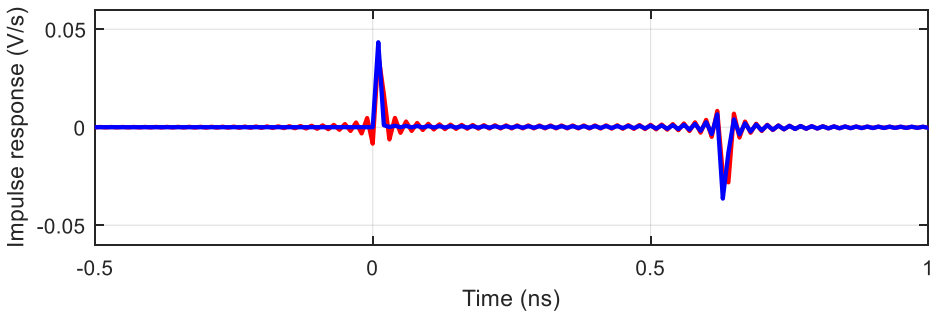


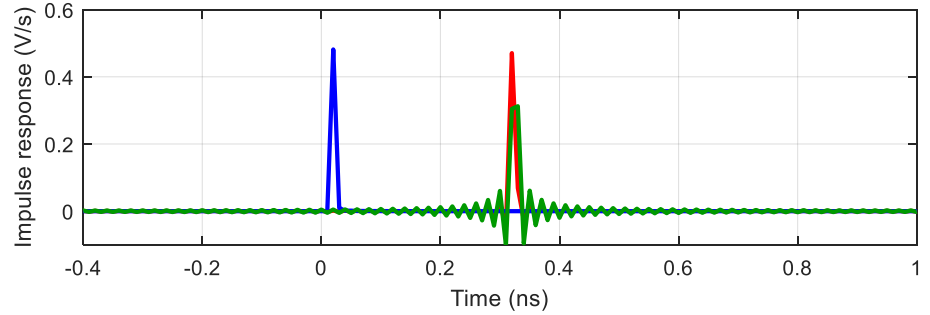


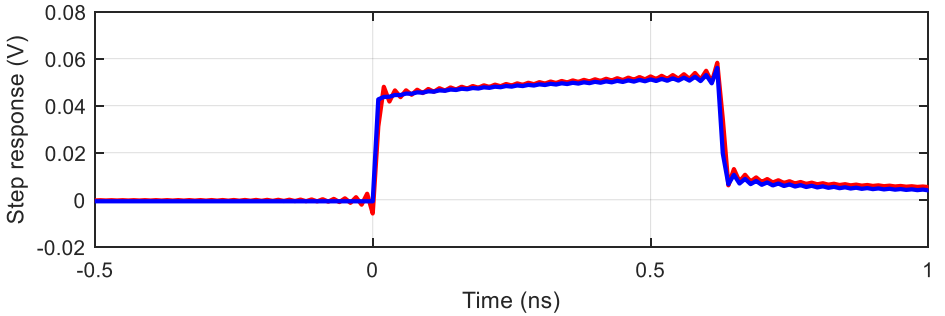
Causality correction methods
Hilbert – amplitude - phase

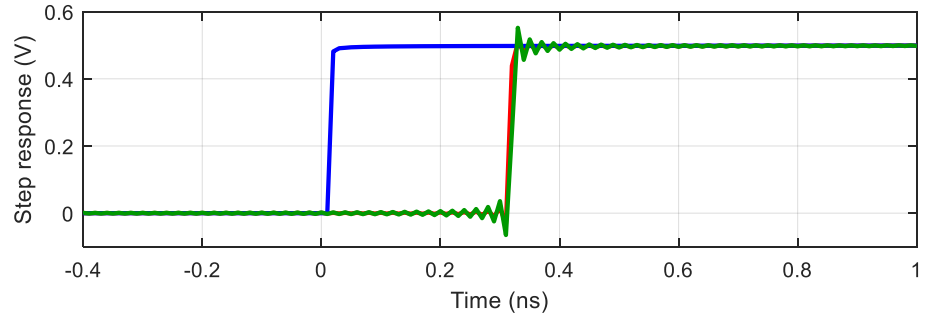




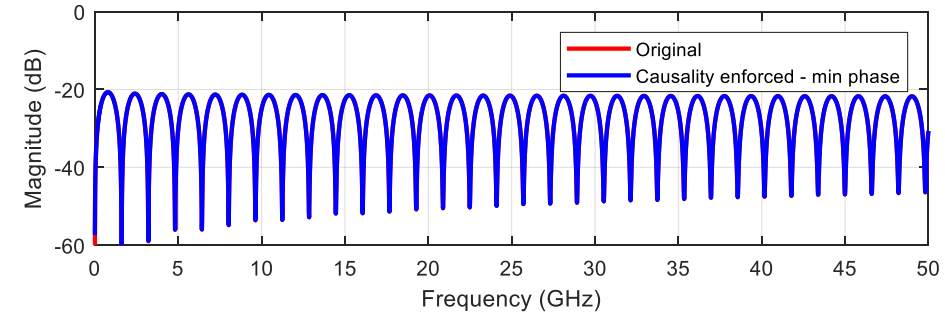


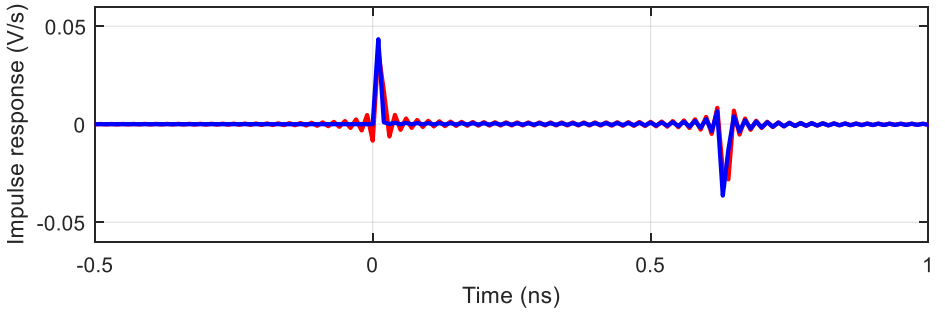


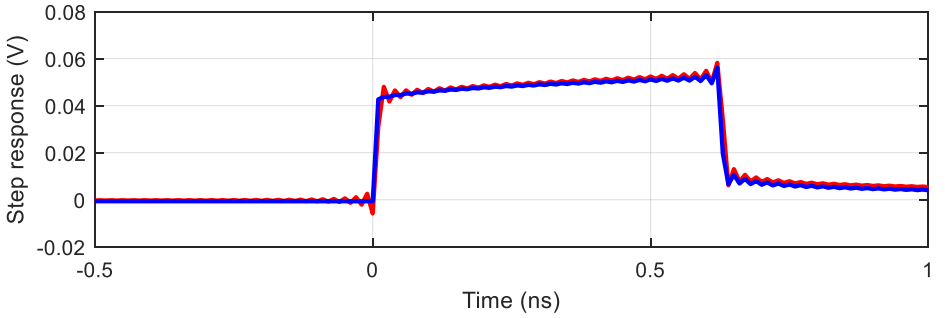


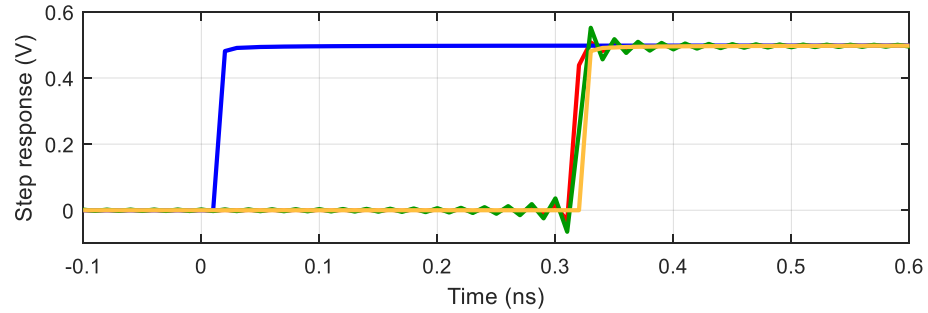


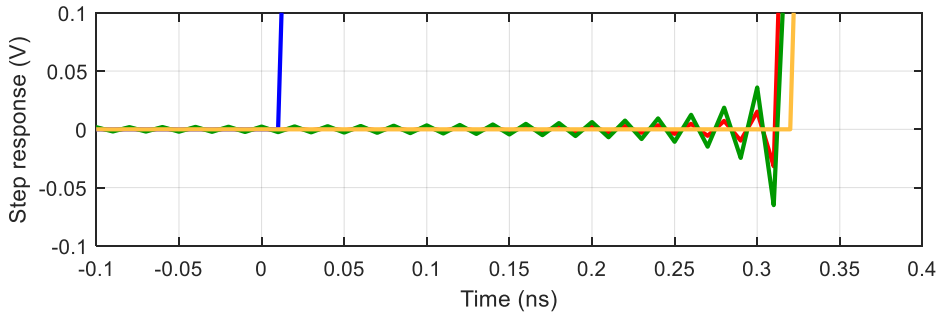
Causality correction methods
Hilbert – amplitude - phase

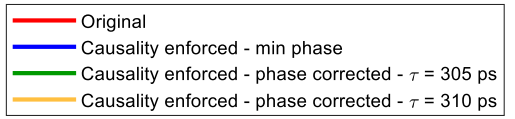






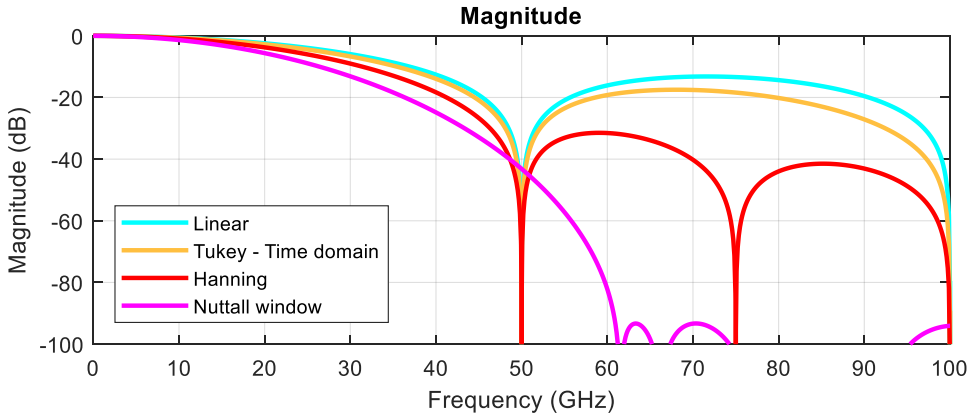


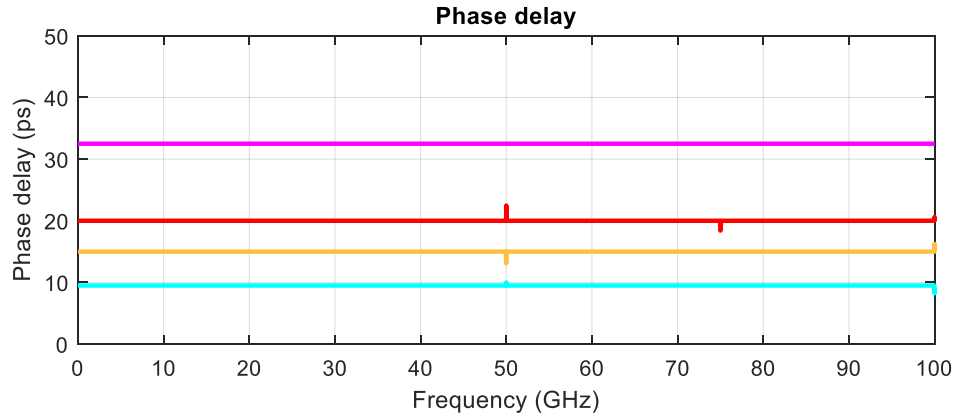


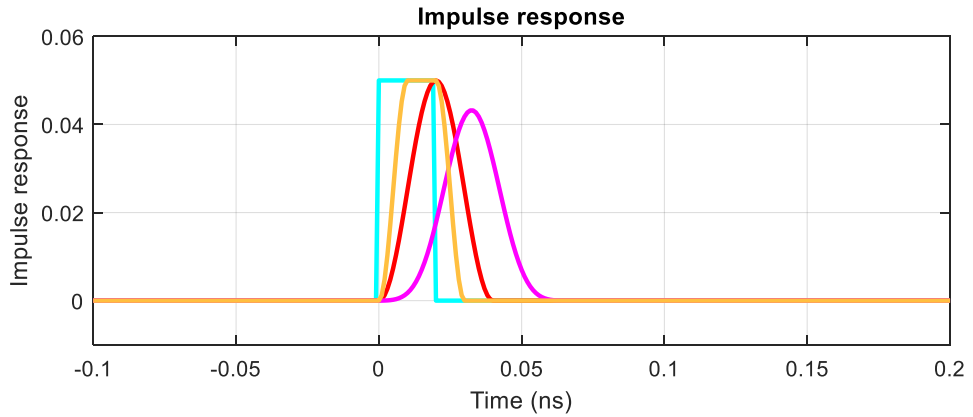


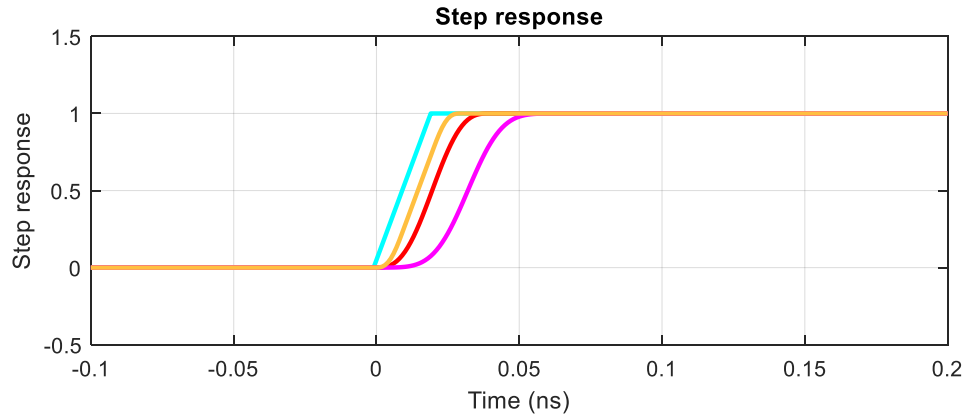
Theoretical filters

Filters
Theoretical filters









Filters
Non causal filters